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NOTES ON
ALTERNATE CURRENTS
FOR STUDENTS

HAROLD H. SIMMONS

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NOTES ON
ALTERNATE CURRENTS
FOR STUDENTS

BY

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PREFACE.

THIS small book consists principally of a portion of a larger work on the outlines of Electrical Engineering which the author has in preparation. The issue of this portion as a separate small book is the outcome of enquiries from numerous students as to where they should find in print the gist of the author's elementary lectures, and of requests that he should publish them. The author has much pleasure in offering to those students part of the section dealing, in the larger work, with alternating currents, in the hope that it may fill a gap and meet a much-felt need. The references to other sections of the larger work have, in most cases, been allowed to stand in the present book.

Sevenoaks,
March, 1904.

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NOTES ON
ALTERNATE CURRENTS.

THE EFFECTS OF UNSTEADY CURRENTS.

THE magnetic effect of a current flowing in a conductor is capable, under suitable conditions, of reacting upon the conductor in a manner which produces results that are very startling at first sight. The suitable condition for the production of this reactive effect is that the current shall not be a steady one, and this condition may be obtained in several ways. If we consider what happens in a coil of wire when the current in it is changed in any way—for instance, when the current is made to grow steadily from nothing up to a fixed value—we see, in the first place, that the magnetic flux produced by that current will also grow from nothing up to a fixed value. If there be no iron in the magnetic circuit, then the magnetic flux will, of course, be strictly proportional to the current in the coil. In the second place, we see that if, when the current has reached its final value, there are magnetic lines in the coil, and there were none to begin with, or even if there were some, but a different number, then we have effected a change in the number of lines threading the coil, and, according to what we have seen in Section 4,* there must consequently have been some cutting of lines of force by the coil and a resulting

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* "Outlines of Electrical Engineering."

E.M.F. Without stopping here to consider how the cutting actually arises, let us investigate what the direction of the E.M.F. will be as compared with the current. Lenz's law (Sec. 4, ch. 4*) shows us that the introduction of a field into a coil tends to generate an E.M.F. of such a kind as will produce a current to oppose the introduced field. Now in the present case we have a current which has grown, and which in its growth has, in effect, introduced lines of force into, and in a certain direction through, the coil; the introduction of these lines must therefore set up an E.M.F. of a kind to produce a current which would oppose these lines, and would therefore be in the opposite direction to the current which we have forced through the coil. This E.M.F. is thus also in opposition to our current, and since it has been induced in the coil by the action of the current itself (through the medium of the field) it is called the back E.M.F. of self-induction. The self-induced E.M.F. plays a most important part in all circuits where the current is frequently changed, either in a circuit in which the current is stopped and started frequently or in a circuit in which the current is continually changing. Circuits in which the current is continually changing are in common use, the currents being called *alternating currents* and the circuits *alternate current circuits*. Before looking into the peculiar properties of alternating current circuits it behoves us to study self-induction a little more closely.

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* "Outlines of Electrical Engineering."

It is important for the student to realise first that self-induction can only be present when a current is changing, and next that when the current is changing self-induction must always be present. Certain circuits, it is true, are spoken of as having no self-induction, or as being "non-inductive," but, as we shall see, it is not that there are no such forces at work, it is simply that the forces at work are all so arranged as to balance one another almost perfectly, so that the ultimate self-induction is negligibly small. Let us examine now a few cases of changing currents, with a view of seeing exactly how each will be affected by self-induction.

One of the very commonest cases of a changing current is that of the electric bell. The ordinary trembling bell automatically stops the current every time the hammer is attracted to the gong, and then when the hammer flies back the current is started again, only to be again broken just as the hammer approaches the gong. We have here, then, a case of a circuit in which the current is continually being started and stopped, and from what has just been said we should expect to find in this circuit the effects of self-induction. As a matter of fact, we do find them, and they usually show themselves in a very familiar effect. So soon as the spring contact leaves the fixed contact, and the current in the coils dies out, the magnetism also dies down; the magnetism is, of course, fairly powerful, and the coils of wire contain a considerable number of turns. The whole magnetic flux in dying down must be regarded as cutting the whole of the turns, and since the dying away of the field occurs very rapidly we have all the necessary factors for a big E.M.F. It will be remembered that the induced E.M.F. is proportional to the *rate of cutting*, and hence the number of turns in the coil, the magnitude of the magnetic flux which dies away, *and the rapidity with which it dies* all influence the magnitude of the generated E.M.F. In the present case we have a very considerable product from these three factors, and, indeed, it is so great that when the two contacts are just separating the weakening of the field is so rapid that the self-induced E.M.F. becomes large enough to make the current jump across the air gap between the two contacts, and we see the resulting spark. We shall find that whenever a circuit is broken and a spark produced, the spark is the result of the high E.M.F. generated by the falling off of the magnetic field. It is now left for us to consider what will be the effect of self-induction on the coil when the current in it is started. The current is started by the closing of the contacts, and this means that the full E.M.F. of the battery is at once put to work on the coils. We are so accustomed to finding our bell respond instantly to the pressing of the bell-push that it is only natural to find it commonly believed that the current instantly rises to its full value. Not only is this idea supported by the common experience with bells, but those who use much heavier currents in engineering work are so accustomed to seeing their instruments respond apparently instantly to the switching on of currents that they, too, apart from their knowledge, would assume that the current instantly rises to its full value. That this cannot be the case in a circuit containing any magnetic apparatus whatever is obvious, for when the switch is closed, and the current begins to rise, the magnetism in the apparatus begins to rise too, and the growing of this field must of necessity set up a back E.M.F. of self-induction, opposing the current producing the field, with the result that the growth of the current is retarded. In most circuits in common use the time taken by the current to grow from nothing to its maximum value, when a steady E.M.F. is suddenly applied, is, indeed, small—it may well be so small as the $\frac{1}{100}$th of a second—so that it is no wonder in such cases that the current is regarded as growing instantly to its full value. In many circuits, however, the effect of self-induction is much greater, and we may easily find the current taking as much as half a second to grow to its full value, whereas a circuit specially designed to have a high self-induction may cause the current to take as much as three or four seconds to grow to its full value. By "full value" is meant a value not appreciably less than $\frac{E}{R}$. In the case of a certain bipolar shunt generator of 20 kw. at 110 volts, the shunt current would take 15·7 seconds to reach 99 per cent. of its $\frac{E}{R}$ value, if 80 volts were suddenly applied to the terminals of the shunt winding.

The whole effect of self-induction may usefully and accurately be likened to the effect of *mass inertia* in mechanics, self-induction in electricity giving to the current an apparent mass. In the study of mechanics we find that whenever a body has its motion changed—that is, when it is started or when it is stopped, when it is accelerated or when it is retarded—we find a force at work opposing the change, a force, moreover, which comes into operation only so long as a change lasts. When a single railway truck has to be moved without the assistance of steam, the men pushing it know well enough that nearly all the labour is connected with getting the truck into motion, and that if the track be level, so soon as the truck has reached the required speed, far less work is necessary to keep it going than was necessary at the start to set it in motion. The men, in other words, had to do a great deal of work in starting the truck, and, having started it, have far less to do in maintaining the motion. The maintenance of the motion only involves the overcoming of friction, whereas the getting into motion involved overcoming the inertia of the truck. The same remarks will, of course, apply if, when the truck has reached one speed, it is desired to increase the speed to a higher value; or if the change to be produced is of the opposite kind, that is, if the truck is to be deprived of its speed, everybody knows that forces due to the inertia of the truck will again come into play. If the truck is to be stopped without the use of brakes (which very much mask the effect of inertia), great force will have to be exerted against the truck, whose inertia opposes this change, and it is very well known that the more rapid the change the greater the force necessary; in other words, the more suddenly the truck has to be stopped the greater the force necessary. As the men have to push against the motion of the truck to stop it, the truck may be considered as pushing them, since they are forced to step backwards, and hence the truck can properly be regarded as doing work on the men, for it is doing precisely to the men what they did to it when they started it.* The truck thus gives out work, and if it be asked where the truck got the work from, the answer is, the men put the work into it when they started it into motion, and that work, the work done in starting, was stored up in the truck by virtue of its motion so long as its motion continued. In losing its motion the truck did but give out again what it had by virtue of its motion. These facts have been dealt with in such detail because they very fully illustrate the facts of self-induction.

The starting of a current in a circuit possessing self-induction involves the starting of a magnetic field, which, as it starts, cuts the conductors of the circuit, reacting upon them and producing a force opposing the start. When the current has reached a steady value the magnetism remains steady, and all that has to be done to keep the current going is to overcome the resistance of the circuit. If, now, the current has to be stopped, the magnetic field dies away, cutting the circuit in the opposite sense, reacting upon it to produce an E.M.F. opposing the stoppage of the current. In so doing, the E.M.F. of self-induction, by tending to maintain the current, does work upon the circuit. The magnetic flux, by virtue of its reactive effect on the coil producing it, represents a definite amount of work done, and the need of this work is felt in the circuit by the current having to be forced against the opposing force of self-induction, and the magnetic field, in dying away, does but restore to the circuit the energy that had been stored in it, this energy appearing as the self-induced E.M.F. maintaining the current, when the original force producing the current had been partially or entirely removed. All the time the magnetic flux was not changed, it neither involved an expenditure nor a return of energy, the only energy required being that necessary to overcome the resistance of the conductors, a thing quite independent of the field. The effect of the self-induction upon a circuit to which a steady E.M.F. is applied is embodied in what is known as Helmholtz's Law, which may be written as follows:

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* That is, it is pushing them against the opposing force which they exert, and hence is delivering energy to them. Of course they, on receiving it, do not to any marked degree store it as the truck did, but dissipate most of it in several directions.

$$C = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

Here C, E, and R have their usual meaning, the quantity e being the base of napierian logarithms, and L a quantity to be described presently, and known as the co-efficient of self-induction of the circuit.

If a curve having the above equation be plotted, we obtain that given in Fig. 1. The quantity $\frac{L}{R}$ is called the time constant of the circuit, and if we examine it we come to the extraordinary conclusion that if it were not for resistance a current in any inductive circuit would take an infinite time to grow to

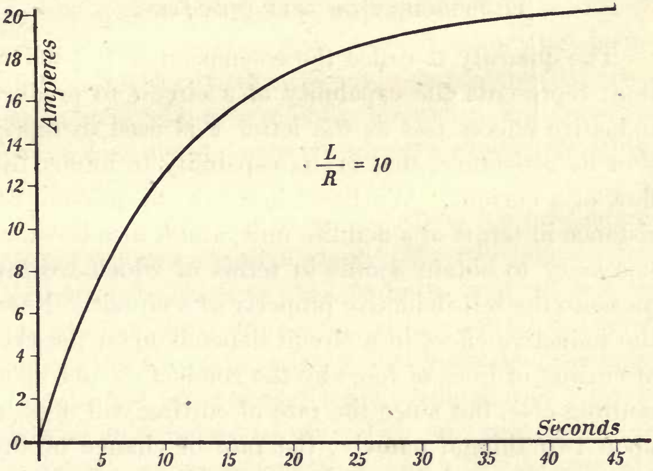

FIG. 1.

any value, that is, it would grow infinitely slowly, and further, that in a circuit of given self-induction, increasing the resistance will enable the current to grow more rapidly to its final value. Resistance, therefore, comes to us in this connection in the guise of a friend, and, indeed, the common attitude of beginners towards electrical resistance in regarding it as an unmitigated evil is as unjustifiable as the corresponding attitude to mechanical friction. Just as we find friction indispensable in mechanics in enabling a screw or a nail to hold, or a man to walk, so also we find resistance in electricity indispensable in enabling us to turn current to a useful purpose.

CO-EFFICIENT OF SELF-INDUCTION.

The quantity L, called the co-efficient of self-induction, represents the capability of a circuit to produce inductive effects just as the letter R is used to represent its resistance, that is, its capability to hinder the flow of a current. We have, however, to measure resistance in terms of a definite unit, and it also becomes necessary to obtain a unit in terms of which we can measure the self-inductive property of a circuit. Now, the inductive effect in a circuit depends upon the rate of cutting of lines of force by the conductors and upon nothing else, but since the rate of cutting will depend upon two things, namely, the rate of change of the magnetic flux and the number of conductors composing the circuit, embracing the flux, we can investigate the self-inductive property of a circuit by considering these two quantities. If we assume that every line threads all the conductors, the number of conductors to be cut by the flux is a very simple quantity, and requires no further consideration. The rate of change of magnetic flux involves, however, two things, one being the total range of the change, and the other the speed with which that range is passed through; we may, for example, change a matter of 2,000 lines in 200 seconds, getting a change of 10 lines per second, or we may get a total change of only 100 lines, which, if it occur in half a second, will mean a rate of change of 200 lines per second, giving an induced E.M.F. 20 times as great as that which the former rate of change would have given. The changing magnetism that we are considering is a magnetic flux produced by the current, and we may hence transfer our attention from the flux to the current, as a rapidly changing current will mean a rapidly changing flux, and a slowly changing current will mean a slowly changing flux.

To fix our ideas, let us consider how the production of a given flux may be effected. We will take a non-magnetic tube, such as glass or brass, and produce in it a flux of given strength. This may be done by winding the tube with wire of any thickness, the only necessity laid upon us being the production of a given number of ampère-turns. Let us provide these ampère-turns first in one way and then in another. If the number required be 250, let us provide in the first case 10 turns of wire carrying 25 ampères, and in the second case 100 turns of wire carrying $2\frac{1}{2}$ ampères. If we compare the relative self-inductive values of these circuits by supposing the current to change at the same rate in each, say, at the rate of half an ampère per second, then if the current is getting less uniformly at that rate it will take 50 seconds to die down to nothing in the first case and 5 seconds in the second case. In the first case, then, we shall have the flux of, say, N lines, cutting 10 turns of wire in 50 seconds, and the total rate of cutting will be $\frac{10 \times N}{50}$ lines cut per second. In the other coil, we shall have $\frac{100 \times N}{5}$ lines cut per second, that is a rate of change in the second case 100 times as great as that obtained in the first case. It must be clearly noted that we have in each case used exactly the same flux produced by exactly the same number of ampère-turns, the only difference in the two circuits being the number of turns of wire composing them. The numbers given above would lead us to suppose the rate of cutting to be proportional to the square of the number of turns, provided the rate of change of the current and the total flux produced remained constant. This supposition is easily shown to be correct by proceeding to discuss, in a general way, mathematically, the particular case given above.

We might proceed on these lines to define the self-inductive property of a circuit and to lay down the magnitude of a unit, in terms of which that property may be expressed. This has been done, a circuit being said to have the unit of self-inductive property if the back E.M.F., of self-induction in it is one volt when the current in it is changed at the rate of 1 ampère per second. This is otherwise expressed by saying that when these conditions hold, the co-efficient of self-induction of the circuit is one unit, and the name "Henry" has been given to the unit in honour of the American electrician Henry. Hence, if a circuit is said to have a self-induction co-efficient of 3 henrys, what is meant is, that if the current in it be changed at the rate of 1 ampère per second, there will be 3 volts back E.M.F. of self-induction. If the tube above referred to were 2 inches long and about $5\frac{5}{8}$ inches in diameter, the fine wire winding would give a co-efficient of about ·02 henry, and the thick winding about ·0002. The henry is a somewhat inconveniently large unit, the co-efficients which have most commonly to be dealt with being of the order of $\frac{1}{100}$th of a henry. On the other hand, the henry is a most convenient unit, because it has been laid down on such excellent lines, all the quantities in it being, as will be noted, of unit magnitude in the scale of the practical units. If, therefore, we know the co-efficient of self-induction, L, of a circuit, we have only to multiply this by the rate of change of current to obtain the magnitude of the back E.M.F. in volts.

In the case discussed above, the conditions were about as simple as it is possible to have them. If, however, our coil had been wound on iron or steel instead of on a non-magnetic material, matters would have been more complex.

In the example given it was assumed that the current, and hence the lines of force, died away at a uniform rate throughout the whole period of that operation. The second assumption was correct in view of the fact that the coils were wound on glass or wood, or some non-magnetic and non-conducting material. If, however, the core be of magnetic material such as iron, steel, or even nickel, the rate of dying of the flux will not correspond precisely with the rate of dying of the current, but will depend upon the degree of magnetisation that has been attained at the maximum, and upon the precise nature of the magnetic core. In other words, the flux produced is not proportional to the current, but follows a very irregular law. This has been pointed out in Section IV.,* and the curve of magnetisation of iron is again given here in Fig. 2. An examination of this curve makes it at once evident that if the current change in a given time so as to reduce the magnetomotive force down any one of the three steps

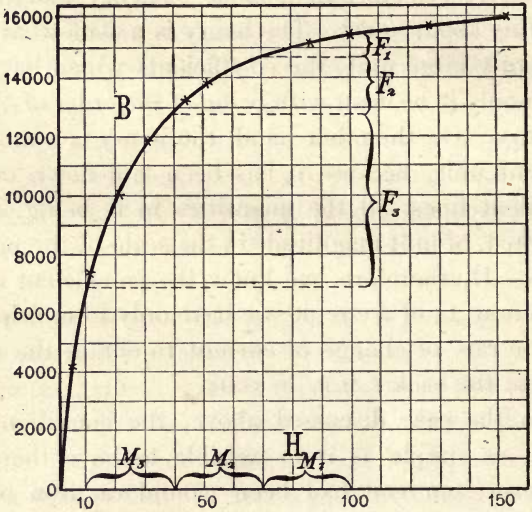

FIG. 2.

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* "Outlines of Electrical Engineering."

marked M 2, M 3, M 4 on the diagram, that any two such diminutions of the magnetomotive force would not be accompanied by equal diminutions of the magnetic flux; hence, if we make the three changes in equal times, although in each case we have the same rate of change of current, we have not the same rate of cutting of lines in the three cases. The rate of cutting of lines, however, is what determines L for this circuit, and hence we see that L is not a fixed quantity for the circuit in question, and the deviation from constancy is due to the variation in the magnetic permeability of iron. This change in the co-efficient of self-induction is of very great importance when the degree of magnetisation of the core is pushed above the knee of the curve; but we see, too, that when it is pushed above that point the value of L remains sensibly constant for

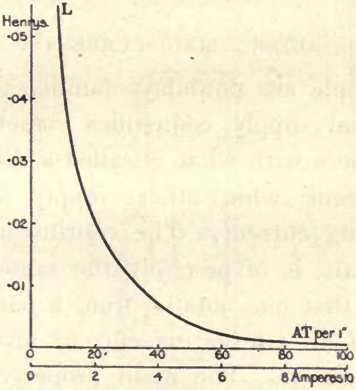

FIG. 3.

any further degree of magnetisation, since the curve of magnetisation above that point is a straight line. The portion, too, lying below the knee is approximately straight for a considerable distance. The accompanying curve, Fig. 3, illustrates the changes in the value of L for a coil wound on a cast steel core, the abscissæ giving the excitation for this particular coil.

ALTERNATING CURRENTS.

Most people are nowadays familiar with the fact that electrical supply companies sometimes provide their customers with what is called a direct or a continuous current, while others supply what is called an alternating current. The continuous current, as is well known, is of precisely the same character as the current that one obtains from a battery, but the properties of an alternating current are not so commonly understood. The main property of an alternating current, as the name implies, is that it continuously reverses its direction. Perhaps the easiest way to convey an accurate idea of an alternating current is to liken it to water in a pipe pumped to and fro by an oscillating piston as shown in Fig. 4. The piston oscillates to and fro in the cylinder from end to end, reversing its direction each time it reaches the end of a stroke, and we may consider the electricity in an alternating current circuit to oscillate round the circuit just as the water in the analogue is caused to oscillate round the pipe circuit. In the figure the piston is supposed to be driven by a crank. This crank must be supposed to revolve at a uniform speed, and it will at once be seen that the motion of the piston in the cylinder is continually changing; in fact, its motion is precisely the same as the motion of a pendulum or swing.*

Now every child knows when it is in a swing that, commencing from the top, when the swing is just about to descend, the motion, beginning at nothing, gets faster and faster until it reaches its maximum speed as it passes between the posts, and the speed then falls off until the highest point is reached on the opposite side. So with our piston. The fly wheel driving it is supposed to rotate at a perfectly steady rate, but the piston will have a speed which depends upon its position, its speed being greatest in the middle of the stroke, and continually changing from that

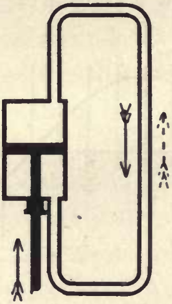

FIG. 4.

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* That is, neglecting the angularity of the connecting rod. The motion is not a simple harmonic motion, unless the connecting rod is infinitely long.

greatest value down to nothing as it reaches the end of a stroke, and then from nothing up to the maximum as it returns to the middle position. This simple matter has been spoken of because it describes a series of operations which is of great importance. The important point about it is that the speed of the piston is always changing. We want to know exactly how it changes. If we plot a curve to show how that speed changes we must proceed first to study the speed itself as follows: Let us draw a semicircle on a vertical diameter

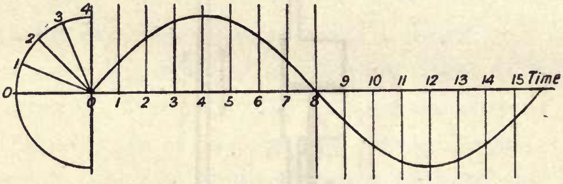

FIG. 5.

(Fig. 5) to represent the path of the crank-pin through half a revolution, and then, if we neglect the very slight error due to the connecting rod being short, we can get the motion of the piston in the following way: From the centre of the semicircle draw a line at right angles to the diameter for time measurements. Now mark off a number of equidistant points on the circumference of the semicircle, and along the horizontal line mark off a number of points which must also be equidistant from one another, but whose distance from one another need have no particular relation to the distance between the points on the semicircle; since it merely represents some interval of time. Now let us suppose our piston to be at the middle of its stroke and moving up; the crank will then be lying horizontal, and the end of it, therefore, will be at the point marked 0 on the semicircle. If the crank revolves clockwise it will reach points 1, 2, 3, 4, etc., in succession, and the time taken to pass from any one point to the next will always be the same. The points along the horizontal may be taken to represent the lapse of time, and if our crank take one minute to complete one revolution, it will take two seconds to pass from point 0 to point 1 on the circle; the distance from 0 to 1 on the horizontal line may then be used to represent two seconds and so on. Now for the position of the piston. When the crank-pin has moved from 0 to 1 the piston will have been raised vertically by the same amount that the crank-pin has risen (always neglecting the angularity of the connecting rod). If we started making our observations precisely at noon the crank-pin would have been at 0 at noon, and the piston in the middle of its stroke, and at two seconds past noon (this instant of time is represented by the point 1 on the horizontal line) the piston will have risen through a distance shown by projecting the horizontal line from the crank-pin. This may be projected far enough to come over the point 1, and this will give us a point representing by its distance above the horizontal line the position of the piston above its middle point, and by its distance from the diameter of the semicircle the time that has elapsed since noon. We proceed to find the other points in exactly the same way. When the crank-pin has reached the point 2 the projecting line will show the height through which the piston has been raised, and the point where this projecting line meets a vertical line through the point representing four seconds past noon will be the next point on our curve. Of course, when the crank has reached the vertical position, its downward passage to the right hand of the diameter will be exactly the same as the return movement through the points 7, 6, 5, and so on, and we get the complete curve shown in Fig. 3. It must be clearly understood that any point on this curve represents the connection between two things, namely, the distance of the piston from its mid position and the instant of time when it is there.

This apparently simple matter has been thus described at length in order that those readers who are not already familiar with it may have ample material to construct such a curve for themselves. Simple as this is, the amount of information conveyed to a student by actually constructing such a curve is invaluable, and no one who has not constructed one can afford to neglect this exercise.

The curve we get shows position and time; what we want, however, is the speed at successive instants. The speed of motion is seen by the slope, the steepness, of the curve, and we can apply this test to the two crests, one above and one below the horizontal line; here, of course, the curve has no steepness at all, it being absolutely horizontal *just at those two points.* Elsewhere we see, starting say from the top crest, that the slope of the curve gets steeper and steeper till it crosses the middle line, that is, the piston moves faster and faster; having passed the middle line the speed gets less and less until the piston comes to rest at the point of reversal. The curve reveals the fact that the motion of the piston is never the same for two successive instants, unless, indeed, one imagines two instants of time inconceivably near to one another at the end of a stroke. It is obvious at such a point, namely, at the end of a stroke, that the piston must come to a standstill in order to reverse, but that, except at these two points at each end of the stroke when it reverses, its motion is always changing.

Now let us consider the bearing of all this upon the alternating current circuit. The curve is, of course, a curve of sines, and it is usual in dealing with alternating current problems to assume that the E.M.F. of the commercial alternating current generator follows the law of sines; that is, that the curve we have drawn would represent the connection between E.M.F. and time, just as well as it represents the connection between piston position and time. The reason for this assumption is that it is extremely difficult to deal with many problems on the assumption of any more complex law, and it is found in practice that the E.M.F. curve of commercial machines does not depart very widely from this particular shape.

In dealing then with alternating currents, we deal with sinusoidal waves of electricity, and our curve may be used to represent current strength plotted vertically against time plotted horizontally. The electricity oscillates to and fro in the circuit just as did the water in the pipe of Fig. 4. We will, however, defer the consideration of the precise way in which the behaviour of the water is analogous to the alternating current until we come to investigate the effect of a capacity. For the present it will suffice to follow the assumption that the *current* curve is a sine curve. It next follows, therefore, that it has the characteristics already pointed out, and the current is thus *always* changing except for two instants of time (the upper and lower crests) in each period. It further follows from this that the effect of self-induction is always at work except at these two instants of time. Now although the current is always changing, like the speed of the piston, we have not yet seen how it changes. Mathematically it is easily expressed now that we have assumed it to follow a sine law. If c_t means the current at any instant, and f the frequency, that is the number of complete periods per second, $c_{max.}$ being the value at the crest, then

$$c_t = c_{max.} \sin. 2\pi f t.$$

and

$$\frac{dc}{dt} = 2\pi f c_{max.} \cos. 2\pi f t$$

the value of $\frac{dc}{dt}$ being the rate of change at the instant t. For those readers, however, to whom this mode of expressing matters is not clear, we may argue as follows:—In our curve, Fig. 5, time is plotted horizontally and current strength vertically; hence, if the current makes a big change in a short time, the slope of the curve during that period will be steep. On the other hand, if the current makes but little change in a lengthy time, the slope will be barely perceptible; thus the slope of our current curve is an indication of its rate of change. If, however, we look at our current curve we find that, so far as slope is concerned, it may have one of three characters. The slope may be like the slope of handwriting, when, as time passes, the current is getting greater. Such a slope is called a positive slope, since with an increase of time we have an increase of the quantity. We then reach the crest of the curve. Just before reaching the crest the slope, of course, gets continually less, until at the very top of the curve we arrive at the little absolutely horizontal portion. It is of no matter how small the horizontal portion is, but it is obvious there must be a portion that is horizontal, since the curve bends over. This is the second character the slope possesses, and here it is clearly nothing, it is horizontal. Passing on we find that the curve again acquires a slope, but of the opposite kind to the one with which we started, and if the former was regarded as positive this must be regarded as negative, since an increase in the time elapsed is accompanied by a *decrease* in the current; in fact, as time continues, the current not only decreases, but reverses and becomes increasingly negative, until the lowest point has been reached, when the slope is once more zero. In character, then, the slope may be positive, zero, or negative, and it may have varying degrees positive or negative. A close and careful inspection of the curve will show, however, at once, that the slope is greatest just where the curve cuts the horizontal line, the axis of time, and the reader may easily for himself* by graphical

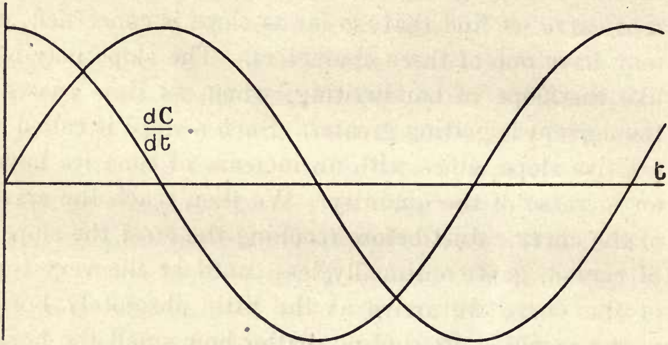

FIG. 6.

means follow out the changes which the slope undergoes in a whole cycle.

All these results are expressed easily and briefly by the mathematical expression given above—

$$\frac{dc}{dt} = 2\pi f\, c_{\max.} \cos. 2\pi f t,$$

and this, again, has been translated into graphics by the curve marked $\frac{dc}{dt}$. In order, now, to find out the magnitude of the back E.M.F. at any instant, we merely have to multiply this quantity, which is the rate of change of the current, by the co-efficient of self-induction L. Or, since the maximum value of cosine $2\pi f t$ is 1, if we are given the frequency we can at once write

---

* See Appendix at end of Section 6 "Outlines of Electrical Engineering."

$$\mathscr{E}_{max.} = 2\pi f L C_{max.}$$

If the permeability remains constant, as is the case when the coil has an "air core," the magnetism is at every instant strictly proportional to the current, and we may therefore draw two curves as in Fig. 7, one showing the way in which the current changes as time goes on, and the other how the magnetism changes. The parts of the curve that lie above the horizontal line indicate that the current is flowing in one direction round the coil, which we will call the positive direction, say clockwise looked at from the left hand end, and where the curve lies below the horizontal line it indicates that the current is flowing in the reverse direction, which we will call the negative. In order to fix our ideas we

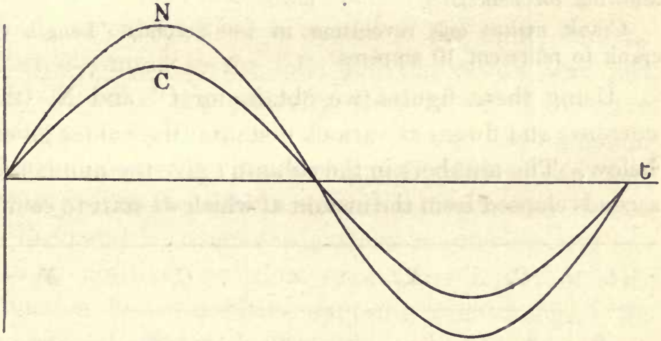

FIG. 7

may consider these two curves to relate to a coil of wire having the following dimensions:

Number of turns ... S = 200 (in two layers)
Outside diameter ... d_1 = 5·05 cm. ... = 1·99″
Inside „ ... d_2 = 6·05 ... ... = 2·38″
Mean „ ... d = 5·55 ... ... = 2·185
Internal area ... ... a = 20 sq. cm. ... = 3·1 sq″
Length ... ... l = 25·4 cm. ... ... = 10″

Total flux per ampère allowing for leakage } $N_1 = \frac{4\pi}{10}\frac{S}{l}a\left(\frac{1-\frac{d}{2}}{l}\right)$ = 176·5

Crank makes one revolution in 180 seconds. Length of crank to represent 10 ampères.

Using these figures we obtain for C^t and N_t (the currents and fluxes at various instants) the values given below. The numbers in the column *t* give the number of seconds elapsed from the instant at which we start to count.

t	C_t	N_t	*t*	C_t	N_t
0	0	0	95	− 1·736	− 306
5	1·736	306	100	− 3·42	− 604
10	3·42	604	105	− 5·00	− 883
15	5·00	883	110	− 6·43	− 1135
20	6·43	1135	115	− 7·66	− 1351
25	7·66	1351	120	− 8·66	− 1531
30	8·66	1531	125	− 9·397	− 1660
35	9·397	1660	130	− 9·848	− 1738
40	9·848	1738	135	− 10·00	− 1765
45	10·00	1765	140	− 9·848	− 1738
50	9·848	1738	145	− 9·397	− 1660
55	9·397	1660	150	− 8·66	− 1531
60	8·66	1531	155	− 7·66	− 1351
65	7·66	1351	160	− 6·43	− 1135
70	6·43	1135	165	− 5·00	− 883
75	5·00	883	170	− 3·42	− 604
80	3·42	604	175	− 1·736	− 306
85	1·736	306	180	0·0	0·0
90	0·00	0	185	+ 1·736	+ 306

If we wish to show how the E.M.F. of self-induction acts we have but to determine the rate of change of the magnetism. This, as we have already pointed out in the case of the current curve, is indicated by the steepness of the curve, the magnetism changing most rapidly where the curve is steepest, and hence we have the two points of greatest rate of change where the curve crosses the horizontal line. Two such successive points are, however, not alike, for one of them is canted to the right, and the other is canted to the left, and the easiest way just now to solve the difficulty thus raised is to have recourse to Lenz's law. The E.M.F. of self-induction always opposes the change of current; indeed, we commonly call it the back E.M.F. of self-induction, hence at the point 0, when the current is growing and becoming positive, we must expect the E.M.F. of self-induction to be negative opposing this change. As the current rises to its positive maximum its rate of rise becomes less, and therefore the back E.M.F. of self-induction becomes less until at the point when the crest is reached, and the rise ceases, the E.M.F. of self-induction is nothing. Now the current begins to decrease, though slowly, and the E.M.F of self-induction still opposing the change must necessarily be positive, and must be of increasing magnitude as the rate of decrease of the current gets faster. Following out this train we arrive at the curve for E.M.F. of self-induction shown at p L C in Fig. 8. Here p stands for 2 πf, and the product p L C gives the height, in volts, of the crest of the curve if C is the height in ampères of the crest of the current curve. Comparing this with the expression just given on p. 32, we find that if p L C is the maximum E.M.F. of self-induction, and p C is the maximum rate of change of the current, L must be a multiplier involving both the number of turns of wire and some quantity

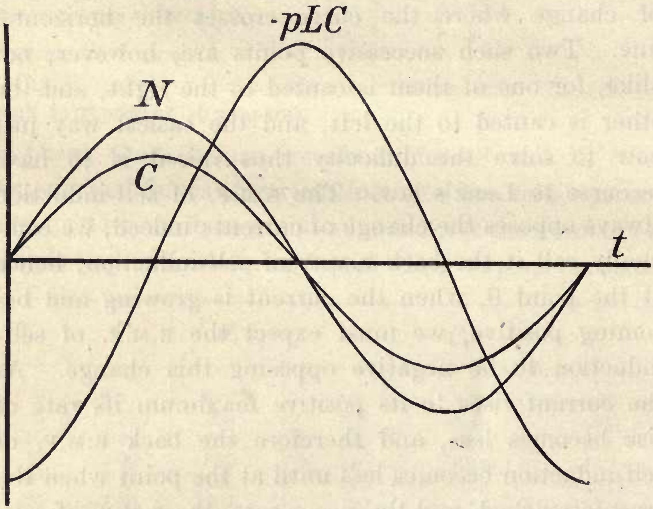

FIG. 8.

referring to the flux, since these two must be involved in the cutting which causes the E.M.F. A little reflection will lead to the conclusion that L is the number of hundred million linkages (of lines and turns) caused by *a current of one ampère* in the coil. In a solenoid then, $\mathrm{L} = \mathrm{N}_{-1} \times \mathrm{S} \times 10^{-8}$. This is clearly the E.M.F. in volts caused by the decay of the flux (and therefore of a current of one ampère) in one second. This agrees with our definition on p. 20. For our coil $\text{L} = 176{\cdot}5 \times 200 \times 10^{-8}$, or 0·000353 henry.

Mathematically the whole of this reasoning may be given as follows :—

$$\text{N}_t = \text{N}_{\text{max.}} \sin. 2\pi \text{f t.} = k\, \text{C}_{\text{max.}} \sin. 2\pi \text{ f t.}$$

$$\mathscr{E}_t = -\frac{d\,\text{N}_t}{d\,\text{t}} \times \text{s} \times 10^{-8} = -2\pi \text{f}\, \text{N}_{\text{max.}} \cos. 2\pi \text{f t.} \times \text{s} \times 10^{-8}$$

$$= -2\pi \text{f} k\, \text{C}_{\text{max.}} \cos. 2\pi \text{ f t} \times \text{s} \times 10^{-8}.$$

$$\mathscr{E}_{\text{max,}} = -\, 2\pi \text{f}\, \text{N}_{\text{max.}} \times \text{s} \times 10^{-8} = -\, 2\pi \text{f} k\, \text{C}_{\text{max.}} \times \text{s} \times 10^{-8}.$$

This last expression shows that the E.M.F. of self-induction depends on four independent things, viz. :—(1) F the frequency, or speed of the crank ; (2) k a factor depending on how the coil is made. It is the area of the coil multiplied by the turns per cm. multiplied by 1·26. (3) C, the maximum current ; (4) S, the total number of turns. That it depends on the rate of rotation of the crank those who have no mathematical knowledge can readily see by bearing in mind that the steepness of the curve indicates its rate of change. If the crank revolved at twice the speed, for example, then there would be just twice as many ripples in, say, five seconds, as there are in Fig. 5, but each ripple would be just as big as before, and if the reader draws for himself a second curve having twice as many ripples he will see that the steepness of the curve is increased, and, indeed, that it is just twice what it was before. Also, if the height were greater in the same period, the like result would follow.

POWER IN AN ALTERNATING CIRCUIT.

To a student accustomed only to direct current working it is somewhat startling to be told that if he reads the ampère-meter and volt-meter on an alternating current switchboard and multiplies the two readings together, he does not necessarily get by their product the watts being delivered to the circuit. This is, however, the case, the reason being that the power that is actually delivered to the circuit depends upon the phase relationship between the pressure and current. The "phase" of an alternating quantity is its condition. Two currents, one of which is growing to a positive maximum, while the other has passed the maximum and is declining, are said to be "out of phase with one another." To be perfectly "in phase" they must perform their operations of reversing, etc., precisely together; they must be perfectly *in step* with one another. The same remarks apply to a voltage and a current. Of course, the ampère-meter and volt-meter say nothing at all about this. Let us consider in the first place how the phase relationship between pressure and current can affect the true power being delivered. Let us suppose first of all that we have the two perfectly in phase, that is to say, that when the pressure is at its positive maximum value the current is also at that value, that the two decrease together, pass through zero at the same instant, reach the negative maximum values together, and so on throughout the whole cycle. The total energy delivered in a whole cycle of operations is, obviously, the sum of all the products of ampères, volts and time taken at every instant throughout the cycle, or expressed mathematically it must be

$$\int_{0}^{2\pi} v \times c \times dt.$$

Now when the two are perfectly in phase both are positive throughout the same period of time, and then again both are negative throughout an identical period of time; hence the product of ampères and volts is at all times positive, and if we know all the products of ampères, volts and time throughout the first half of a period, we may double it and obtain an accurate result for the product over the whole period. This is shown in Fig. 9, which refers to a current of 20 ampères maximum, exactly in phase with an E.M.F. of 100 volts maximum. The curve marked W gives the watts at every instant to the scale of kilowatts, and the energy delivered in one period is the height of the watt curve multiplied everywhere by time: this is given by the area included between the W curve and the axis of *time*. Next let us consider the case in which the current has become displaced in the circuit with reference to the E.M.F. by a quarter period, the maximum displacement that is ever attainable, and only so, strictly speaking, in theory. During the first quarter period, as is at once seen by reference to Fig. 10, the E.M.F. is positive while the current is negative, and hence the products throughout this period of time will all be negative. During the ensuing quarter period the E.M.F. will remain positive, and the current has changed to positive also, hence their products will also be positive. In the next quarter period the current is positive and the E.M.F. negative, hence the products are negative; while in the final quarter period of our cycle the current is again negative, while the E.M.F. also remains negative, giving for this portion of the cycle positive products. These results are embodied in the diagram Fig. 10, from which we see that if a whole period or cycle be considered, it may be regarded as consisting of four portions, two of which give positive values for

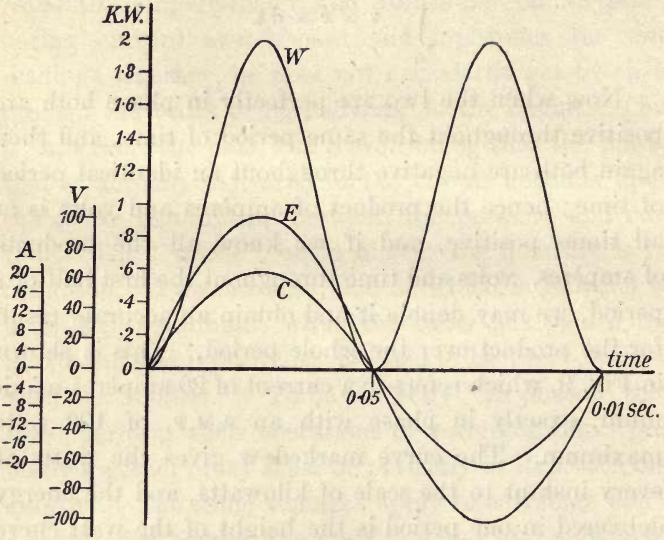

FIG. 9.

the power and two give negative. Moreover, it is at once obvious from the symmetry of the diagrams that the sums of the products in all four quarter periods are identical, and when multiplied by two time give four equal areas. Hence, if we add them all up, their total area will be zero, which leads us to the at first sight surprising result that we may have an alternating current in a circuit, and, of course, an alternating E.M.F.

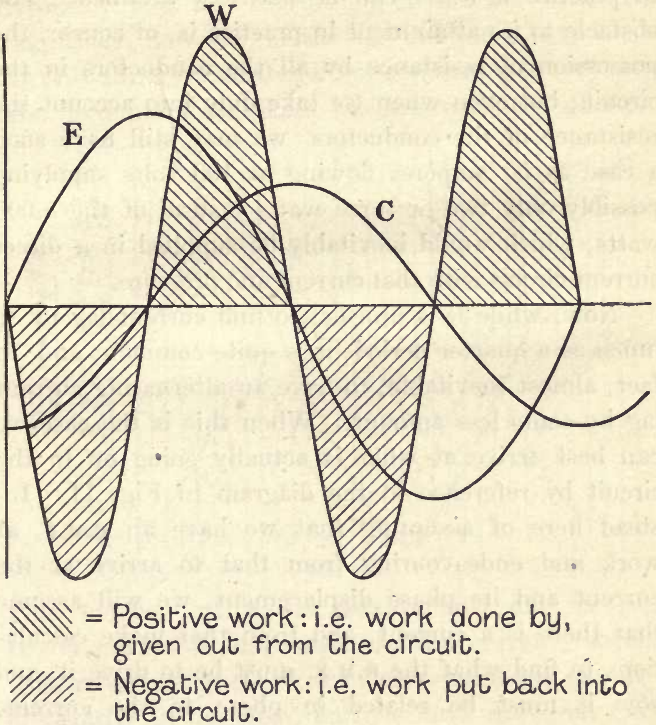
W
E
C
▧ = Positive work: i.e. work done by, given out from the circuit.
▨ = Negative work: i.e. work put back into the circuit.

FIG. 10.

producing it, and yet no power whatever being delivered to the circuit. As, however, we have stated above, this condition is only attainable theoretically; in practice it never can be perfectly attained. The obstacle to its attainment in practice is, of course, the possession of resistance by all the conductors in the circuit; but even when we take duly into account the resistance of the conductors we may still have such a case as 50 ampères flowing at 100 volts supplying possibly only two or three watts instead of the 5,000 watts, which would inevitably be supplied in a direct current circuit with that current and pressure.

Now, while it is unusual to find current lag by so much as a quarter period, it is quite common, and, in fact, almost inevitable, to have an alternating current lag by some less amount. When this is the case we can best arrive at what is actually going on in the circuit by reference to the diagram in Fig. 11. Instead here of assuming that we have an E.M.F. at work and endeavouring from that to arrive at the current and its phase displacement, we will assume that there is a current, and from that make calculations to find what the E.M.F. must be to drive it, and how it must be related in phase to the current. The curve marked C represents then an alternating current which we suppose to be flowing in a circuit that is possessed both of inductive properties and resistance. If the circuit possesses inductance, the power, that is, of producing magnetic lines, it will have the property of self-induction (see page 10), and if we know the details of the circuit we can calculate the co-efficient of self-induction and hence the magnitude of the self-induced E.M.F., when we are given the frequency and the magnitude of the current. The magnitude, however, of the self-induced E.M.F. is not the only thing we need to know concerning it; we must take account of its phase relation with the current.

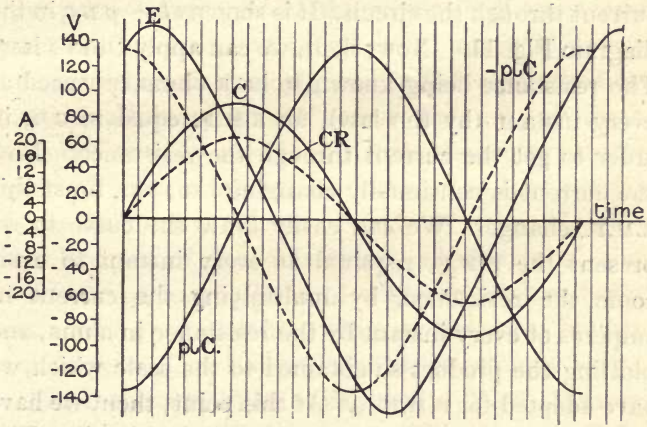

FIG. 11.

This induced E.M.F., as was pointed out on page 10, is always in opposition to the current, so that with reference to the flow of current in the circuit it is negative in its effect, and we must draw our curve accordingly as shown at p L C in Fig. 11. The induced E.M.F. in the circuit is a new factor of which Ohm's law took no account, and if we wish to restore the conditions to which Ohm's law applies we must introduce an E.M.F. which at every instant will effectually nullify that one which is the effect of self-induction. In other words, if we introduce an E.M.F. which at every instant is equal and opposite to the E.M.F. of self-induction, these two will cancel one another, and we may then consider the circuit as being free for the operation of Ohm's law as ordinarily written. Let us then introduce into the circuit such an E.M.F., regarding it as being a part of the requisite E.M.F. to be supplied to drive the current through the circuit. It is shown at $-p\,L\,C$ in the diagram Fig. 11. Now, then, we can apply Ohm's law. The resistance being known to be R ohms, we need at every instant (by the law), an E.M.F. equal to $C\,R$, in order to get the current through the resistance. Now the current is continually changing; so, too, must this E.M.F. change. We can easily draw the curve to represent the E.M.F. required at every instant to overcome the resistance, by multiplying the current in ampères at every instant by the resistance in ohms, and plotting the product so obtained to the scale which we have adopted for E.M.F.'s. At this point, then, we have four distinct quantities to consider in the circuit. The current first, then the E.M.F. of self-induction, thirdly the E.M.F. we have introduced to nullify this, and finally the E.M.F. which we conclude is necessary to overcome resistance. These three are lettered (1) C, (2) $p\,L\,C$, (3) $-p\,L\,C$, (4) $C\,R$. Nos. (3) and (4) have to be supplied, and if we wish to know what is wanted at every instant we must add Nos. (3) and (4) together. If we do this on the diagram by adding the ordinates of (3) and (4) at several points we shall get a fifth curve, as is shown in the diagram at E.

A slow and careful consideration of these curves, and the simple reasoning by which they have been arrived at, is of very great value to a student when first entering upon the study of alternating circuits. For further graphical work, however, the curves are too cumbersome. They are, indeed, almost indispensable in approaching the subject owing to the exceedingly clear picture they give of the sequence of the several quantities involved, but they become unwieldy directly the circuit becomes complicated and directly rapid calculations are required. Of course, they may always be dispensed with by the aid of a little elementary mathematics, if such mathematics include the easy portions of the differential calculus. Indeed, a knowledge of the elements of the differential calculus is of enormous service to an electrical engineer, and will carry him a very great distance, while on the other hand the acquirement of such knowledge is exceedingly easy.

An inspection of the diagram in Fig. 11 reveals the following interesting facts. The diagram differs from those shown in Figs. 8 and 10 essentially by the introduction of the portion marked C R. In the latter we had a circuit supposed to possess a negligible amount of resistance, and hence the C R curve was properly omitted, and we had a current lagging by a quarter period. In the present case the current lags behind the E.M.F. by an amount less than a quarter period. This lag, whatever its amount, is clearly due to the necessity of giving the E curve a factor which nullifies p L C, and hence the lag is due to self-induction in the circuit. Further, this factor (− p L C) increases the total E above what it would be if C R only had to be supplied. That is, for a given C, E has to be greater than it would have to be if self-induction were absent; conversely, therefore, the presence of self-induction diminishes the current. If it were absent the current would clearly be perfectly in phase with the E.M.F. Thus self-induction in a circuit supplied at a given E reduces the amplitude of the current and causes it to lag. By reference to the following diagrams, Fig. 12, we see that without altering the magnitude of the curve p L C we can alter the position of the resultant curve E by altering the amplitude of the curve C R. If we increase the C R curve in magnitude we bring the resultant curve E more nearly into phase with the current C, whereas the smaller the C R curve the more nearly does the amount of lag approach to a quarter period. If, however, we had kept the C R curve constant and varied the magnitude of the p L C curve we should have found that the greater the p L C curve the more the lag, and *vice versâ*. These operations alter the magnitude, of course, of the E curve as well as its position. If we vary both simultaneously we again alter the total magnitude of the E curve as well as its phase position; but it would appear that if we alter the C R and the p L C curves simultaneously so as to get the same resultant magnitude of the E curve, in every case the phase of the E curve with regard to that of the C curve would be determined by the ratio of the C R to the p L C curve. This is easily shown to be the case when

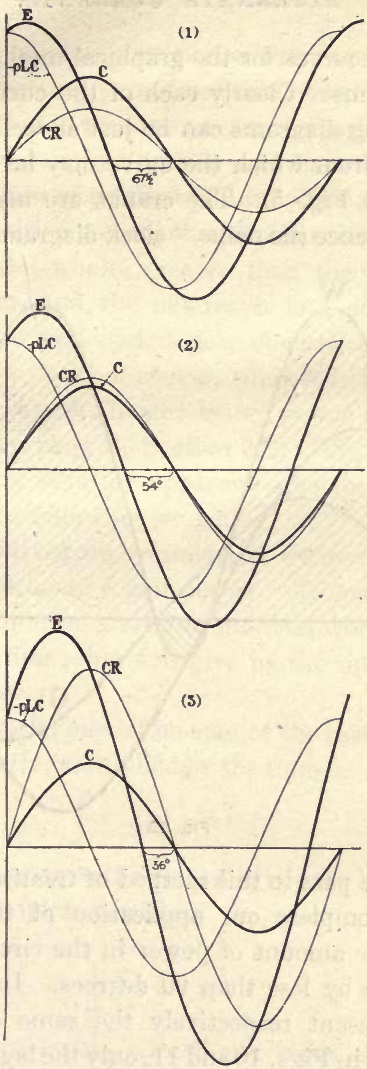

FIG. 12.

we abandon curves for the graphical method known as clock diagrams. Clearly each of the curves shown in the preceding diagrams can be just as well represented by a crank from which the curve may be developed as was done in Fig. 5. The cranks are made to rotate clockwise, hence the name "clock diagrams." Before, however, we pass to this method of treating the subject we must complete our application of the curves to elucidate the amount of power in the circuit when the current lags by less than 90 degrees. In Fig. 13 the curves represent respectively the same quantities as were plotted in Figs. 10 and 11, only the lag here, instead of being 90 degrees, is about 30 degrees. This diagram

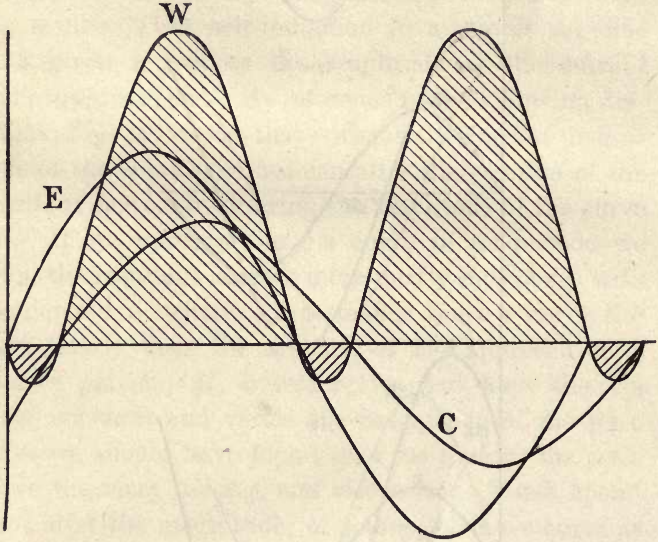

FIG. 13

shows that in one complete period there are, as before, four distinct periods, two of positive power and two of negative, but that in the present case the periods of negative power are less in duration and have a less total height than those of positive power. The area, therefore, under positive power, representing the total work done, is, on the whole, greater than the area under negative power, and the net result is a positive area representing the total work done in one period. An area here represents work, of course, since it is the product of height and length, height being power and length being time (see page 8, Section 1*). The total area when the lag is ϕ is the apparent area multiplied by cosine ϕ, by the following reasoning.

Since the curves are symmetrical we need only consider what goes on in a half period. To get the mean power in this time we must find the total work done and divide by the time; this will give us the mean rate of working, or power.

The total work done is the sum of the instantaneous values of the watts, multiplied by the time or

$$\int v \times c \times d\,t.$$

The work done in half a period if τ is the number of seconds for a complete period is

$$\int^{\frac{\tau}{2}} v\,c\,d\,t.$$

We assume $v = E_{max.} \sin. \theta$, and since the current lags by ϕ, $c = c_{\text{max.}} \sin. (\theta - \phi)$. Substituting these values and dividing by $\frac{\tau}{2}$ to get the mean rate of working we have

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* "Outlines of Electrical Engineering."

$$\text{power} = \frac{2}{\tau} \int_{0}^{\frac{\tau}{2}} E_{\text{max.}} \sin.\, \theta \; C_{\text{max}} \sin.\, (\theta - \phi)\, d\, t.$$

Now, putting angular limits for time limits we see that half a period is π, and that $d\, t = \frac{1}{2\,\pi\, f} d\theta$. Also in t seconds the angle $2\, \pi\, f\, t$ is swept out, therefore the angle π is swept out in $\frac{\pi}{2\,\pi\, f\, t} t$ seconds; this is the time $\frac{\tau}{2}$ seconds, hence

$$\frac{\frac{\tau}{2}}{t} = \frac{\pi}{2\,\pi\, f\, t} t \text{ and therefore } \frac{2}{\tau} = 2\, f.$$

The equation for power must then be written

$$\text{Watts} = 2\, f \int_{0}^{\pi} E_{\text{max.}}\; C_{\text{max.}} \sin.\, \theta \sin.\, (\theta - \phi) \frac{d\theta}{2\,\pi\, f.}$$

$$= \frac{E_{\text{max.}}\; C_{\text{max.}}}{\pi} \int_{0}^{\pi} \sin.\, \theta \sin.\, (\theta - \phi)\, d\theta.$$

$$= \frac{E_{\text{max.}}\; C_{\text{max.}}}{2} \cos.\, \phi.$$

Cos. ϕ is given the name "Power Factor." The presence of 2 in the denominator will be explained under the heading "Virtual Values of Alternating Quantities" (p. 89). Those readers who do not follow the above reasoning will find the explanation on p. 55 very much simpler.

CLOCK OR VECTOR DIAGRAMS.

We have already made use of a rotating crank to exemplify the sequence of phenomena in an alternating current circuit, and we shall now proceed to carry the use of such diagrams further. Diagrams of this kind have been in use some fifteen years, having been elaborated about 1889 by Mr. Blakesley, who gave them the name of clock diagrams. It may be well to remark here that in American treatises cranks are usually taken to revolve counter-clockwise, since that is the direction which is considered positive in works on trigonometry, and in the vector and rotor calculus. In English works, however, it is the almost invariable custom to give the cranks a clockwise direction of rotation following Mr. Blakesley, and we shall, of course, adhere to this practice.

The great value of the clock diagrams lies in the fact that we can indicate any one of the various quantities which are changing by a line which may be supposed to represent a crank. If we give these lines their proper position in relation to one another, we can get from the diagram at once all the information which we want, and it is by no means difficult to give the cranks their proper relative positions.

Directly we convert the curve diagrams which we have just been considering into vector diagrams, the actual angle of lag, by which the current is behind the E.M.F. stands out much more clearly before us. In order to ascertain the angle of lag in the curve diagrams it was necessary to have reference to the axis of time in the first place, and then to remember that one complete period was equivalent to 360 degrees.

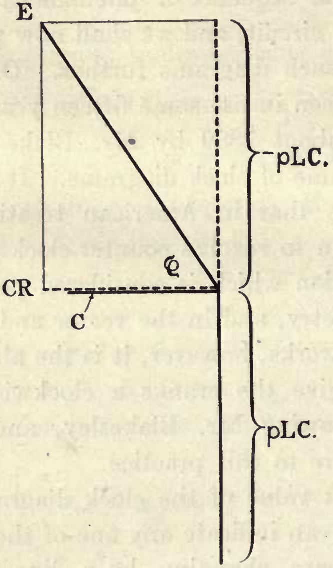

Fig. 14.

The clock diagram, Fig. 14, is arrived at by precisely the same reasoning as the curve diagram, but the alternating quantities have their maximum values represented by the lengths of cranks, all the cranks revolving about a common centre at a uniform velocity, and making as many revolutions per second as we have alternations per second. We will now build up a vector diagram to display the same facts as the curve diagram, Fig. 11. We will begin with the current, which is represented by the crank C. We can choose, of course, any scale whatever for the measurement of current, for example, $\frac{1}{8}$ inch to the ampère, or any other convenient scale that may occur to us. The cranks all revolve clockwise, or right-handedly, and since the back E.M.F. of self-induction follows, as we have already seen (page 36), a quarter of a period behind the current, we must represent it by a crank 90 degrees behind the current crank. It is shown in the diagram by the crank p L C. The E.M.F. necessary to drive the current through the resistance of a circuit is given by a crank which is in step with the current and whose length must be equal on the volt scale to C R. The generator now then must supply, firstly an E.M.F. to neutralise p L C, and secondly it must supply C R. Obviously the voltage necessary to neutralise p L C will be represented by a crank of equal length and 180 degrees ahead of it. This is shown by the dotted line $-$ p L C. We thus have the two components $-$ p L C and C R, which give us all the information we want to enable us to say what precisely is the E.M.F. required. The cranks represent two forces, and we may compound them like any other forces by making a parallelogram and finding the diagonal. The diagonal is shown at E, and represents the E.M.F. that must be supplied to the circuit to get that current C through the resistance and self-induction. The length of the crank E denotes the magnitude, and its position in relation to the crank C denotes the phase of the E.M.F. The student must clearly understand that these cranks are all supposed to be revolving continuously, and in making our diagram we have assumed that it is in our power suddenly to arrest matters at some desired instant while we proceed to portray by the above reasoning the various relationships in question. It is, of course, quite immaterial at what particular instant we arrest the revolving cranks; we should always get precisely the same numerical result, the only difference that would arise from making our investigation at different instants would be in the actual position of the cranks in space, their relative positions would remain unaltered so long as current resistance and self-induction are the same. A very good mental picture of what is going on in the circuit may be obtained by imagining what would happen with the following arrangement. Let a number of glass tubes filled with different coloured liquids be mounted on a very slowly revolving axle, in such a way as to represent the various cranks shown in the diagram, and let the coloured shadows of these tubes be thrown by a strong light on to a screen placed parallel to the axis about which the tubes rotate. Now if the axle be very slowly rotated the shadows of these coloured tubes on the screen will rise and fall in regular succession above and below a horizontal line given by the shadow of the axle, the heights of the shadows above the axle representing positive quantities, and their lengths below the axle negative quantities. Thus we might have a tube filled with red liquid for the current crank, one with blue liquid at 90 degrees behind it for the self-induction E.M.F., a pale blue one to represent the E.M.F. opposing it, a black one for the C R crank, and a white one to represent the total E.M.F. to supply the circuit.

We have seen already (page 4) that when a current lags 90 degrees behind an E.M.F. the power transmitted in a complete cycle is 0, and that, whatever the angle of lag, the power transmitted is given by the product of current and E.M.F. multiplied by the cosine of the angle of lag.

In this diagram, Fig. 14, we see that the angle ϕ between the C crank and the E crank is determined by the relation between the two components C R and $-p$ L C. The need of $-p$ L C throws the E crank in advance of the C R, and hence also of the C crank. We usually say it makes the current lag behind the E.M.F.

Now, if we consider the two components of E, we see one ($-p$ L C) has a 90 degrees phase displacement in advance of C, and hence only represents surging power with no balance in favour of or against the circuit. The component C R is, on the other hand, right in phase with C, and thus represents power to the full. The magnitude of C R is clearly E cos. ϕ, and hence $C \times (E \cos. \phi) = C^2 R$, or the power delivered by the current in heat. If in addition to C R there were in the circuit an E.M.F. exactly in step with the current, but exactly opposite to it, we should have to supply an equal and opposite E.M.F. to nullify it, and this would be in phase with the current. It would, therefore, also represent power without any deduction.

In the light of clock diagrams, therefore, cos. ϕ appears as the factor by which we must multiply the total E to find the component in phase with the current and which is delivering power.

If the angle be less than 90 degrees the power transmitted has a finite value, although exceedingly small if the angle approaches anywhere near 90 degrees. In practice we find a load which does very nearly fulfil this condition when a central station is supplying a network of conductors to which the primary circuits of a number of unloaded transformers are connected. There will be then a fairly large current at a high voltage delivered from the station, but the angle of lag will be so great that the power transmitted is very nearly nothing, so small, indeed, as to be nearly negligible. It is most unfortunate that when this first came to be recognised the current so flowing from the station and representing little or no power came to be called a "Wattless Current." There is no more reason, in point of fact, for saying that the current is wattless than there is for saying that the E.M.F. driving it is wattless. Since the E.M.F. of supply is kept constant, there is a subtle temptation to the beginner to regard the power delivered as the property of the current only, whereas watts are the product of E.M.F. and current, and it is absurd to suppose that they are due to one more than to the other. Moreover, the term "wattless current," besides being less accurate than "wattless E.M.F.," has introduced a great deal of difficulty to students in approaching this matter. In the first place, it is quite impossible to have a truly wattless current flowing in any circuit, or to express it more precisely, it is impossible to have a current flowing in a circuit without the expenditure of some power. This was duly recognised, and in order to get over the difficulty it was said that the current might be regarded as being made up of two components at right angles to one another, one of which—a small one—was in step with the E.M.F. and the other 90 degrees behind it. Of these two components the small one was the "power current," and the one which lagged by 90 degrees was called a truly wattless current. The explanation is all very well as a piece of fine imagination or as a mathematical device, but is utterly misleading as regards physical facts. As a result of the phraseology, language was commonly used which appeared to convey the idea that there were two currents in the circuit, the power current and the idle current, and, of course, the bare idea of two currents at one time in the same circuit is altogether contrary to accepted theory; hence the term wattless current has been the source of much difficulty and obscurity to students. A far preferable way of treating the circuit, in the writer's opinion at least, is to adhere to the theoretically correct assumption of only one current in the circuit, and to split up the E.M.F. rather than the current, into two components at right angles to one another. The presence of two, or even of many E.M.F.'s simultaneously at work in the same circuit is neither theoretically incorrect nor foreign to experience, and is on these grounds alone superior to the old method. We can then speak of the applied E.M.F. leading ahead of the current, and of its being resolved into one component, in step with the current, representing power, and another component exactly 90 degrees ahead of the current and representing no power. The only change in our phraseology needed then is "wattless E.M.F." This wattless E.M.F. very accurately and truly represents the physical facts, it merely stands for the E.M.F. in the circuit which is entirely occupied in coping with the self-induced E.M.F. This latter, as we have already seen, is associated with a magnetic field, which field, on the whole, costs nothing in power, but acts precisely as a spring taking in and storing work when it is compressed, and giving out that same work as it expands, a wattless operation when carried through any complete cycle. So the field, apart from any iron which conducts it, takes in work in its formation, but restores it perfectly when it dies away; the alternations of field causing a surging of energy out of and back into the electric circuit; a wattless operation when carried through a complete cycle. (See Fig. 10, p. 41). The writer has always found that students readily accept and grasp this mode of presenting the subject, while the mere mention of a wattless current invariably throws their minds into confusion in view of the fact, well known to them, that no conductor is devoid of resistance; from which fact they at once, and correctly, argue, that heat must be produced by *every* current, and that there can be, therefore, no such thing as a wattless current.

IMPEDANCE.

So long as we dealt with continuous currents we needed only to consider one property of the circuit, namely its resistance, so that if this were known and the E.M.F. were known we had merely to divide the latter by the former and to find the value of the current. We have just seen, however, in the preceding pages, that the alternating E.M.F. applied to the terminals of a circuit may have to contend with more than simply a resistance; part of it may, in fact, be absorbed in dealing with the opposing effect of self-induction. The magnitude of this amount is given by the expression $2 \pi f L C$ (see p. 33), while the amount that is necessary to overcome resistance is, of course, $C R$, and, as we have just seen, two E.M.F.'s of these magnitudes respectively must be at work at right angles to one another. Both magnitudes are proportional to the current, and hence we shall not disturb their relation in any way by cancelling the letter C from both terms, and we thus find that we have apparently to deal with a circuit without any current in it, but possessed of two distinct properties, that just represented by R, which we call resistance, and the property which is represented by $2 \pi f L$. The letter L we know; it has already been described as representing the co-efficient of self-induction. The other quantities are mere numbers, f being the frequency of the alternations of the circuit in question. To this term $2\pi f\text{L}$ the name Inductance has been given. Let us for a moment stop to consider what it is that we mean by the resistance of a circuit, or rather let us consider what the resistance may be taken to mean. The resistance of a circuit is that property of the circuit which expresses numerically the magnitude of the E.M.F. in volts required to send one ampère through it (assuming that it possesses no other property such as inductance or reactance), and we may thus regard the term resistance as almost synonymous with an E.M.F. Again, the property which we call the inductance of the circuit gives us the magnitude of the E.M.F. required to drive one ampère through the circuit, taking into account its self-inductive action *only* and neglecting for the moment any resistance or other property. We may thus say that the resistance of a circuit gives us the voltage necessary to get one ampère through it taking resistance only into account, and the inductance of a circuit gives us the voltage necessary to get one ampère through it taking inductance only into account, but that the volts necessary to get an ampère through a circuit possessing resistance and inductance is given by the diagonal of a parallelogram, one side of which represents resistance and the other inductance. The magnitude of this diagonal will clearly be

$$\sqrt{\text{R}^2 + 4\pi^2 \text{f}^2 \text{L}^2}.$$

and since it represents numerically the voltage required to drive one ampère through the circuit taking both properties into account, it should be given a name ending in "ance," to make it correspond with resistance and inductance. The name given to it is "impedance." In the diagrams and explanations that we have hitherto been considering we have attacked the

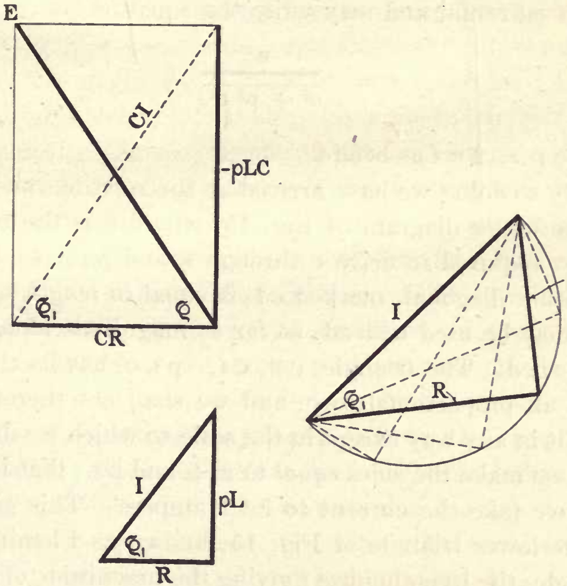

FIG. 15.

circuit from the point of view of current, that is to say, we have assumed a current to be flowing, and have then estimated what E.M.F. was needed. We now, then, have a weapon which enables us to reverse the mode of attack, and to say that if we are given the

E.M.F. at work we can calculate the current. We can do it because we know that the impedance represents the voltage wanted to get one ampère through the circuit, and if there are C ampères we shall want C times as many volts, and we may hence use impedance for alternating current just as we used resistance for direct currents, and may write the equation

$$C = \frac{E}{\sqrt{R^2 + p^2 L^2}}.$$

where $p = 2 \pi f$ as before.

By so doing we have arrived at the relations shown in the upper diagram of Fig. 15, where E is the total E.M.F. required to drive C through R and p L. Clearly the other diagonal, marked C I, is equal *in length* to E, and may be used instead, so far as magnitude alone is concerned. The triangle, C R, C I, - p L C, has its three sides all proportional to C, and we shall not therefore alter it in any way except in the scale to which we draw it, if we make the sides *equal* to R, I, and p L; that is to say, we take the current to be 1 ampère. This gives us the lower triangle of Fig. 15, known as Fleming's triangle, the hypothenuse I giving the magnitude of the E.M.F. to send *one ampère* through the circuit. Now, although the side I is not in the proper phase relation to the other two, when considered in this light, it gives us what we want. If we have a rectangular parallelogram it is clearly immaterial which of the two diagonals we take; they will both be equal in length, and if direction is of no importance either will serve. In constructing vector diagrams direction was of importance, because we wished to represent the angle of lag accurately, and we consequently took the diagonal which divided the angle between the resistance and inductance. Fleming's triangle is composed of resistance and inductance and the other diagonal of the parallelogram, which in magnitude, of course, represents impedance (see Fig. 15). Moreover, since the parallelogram is a rectangular one, the angle ϕ_1 is also the angle of lag; hence Fleming's triangle at once enables us to calculate graphically the magnitude of the impedance and the amount of the angle of lag without actually completing the parallelogram. This part of our subject must, however, not be left without reference to a further use of this most valuable construction. Fleming's triangle is very useful when we wish to adjust the proportions of inductance to resistance without altering the impedance. All we have to do is to draw a semicircle on a line whose length represents, to scale, the impedance we want. The semicircle is the locus of the right angle of all possible right-angled triangles having the diameter for hypothenuse. Fleming's triangles are, of course, right-angled always, and if a chord is laid in the semicircle, of a length to equal pL (to the same scale as that for impedance), the chord to the same point from the other end of the diameter gives the value of R. We see at once how to adjust matters to get the desired current at any given ϕ.

CAPACITY.

Another property of the electrical circuit which, in addition to self-induction, only comes into play when the current is changing is the property known

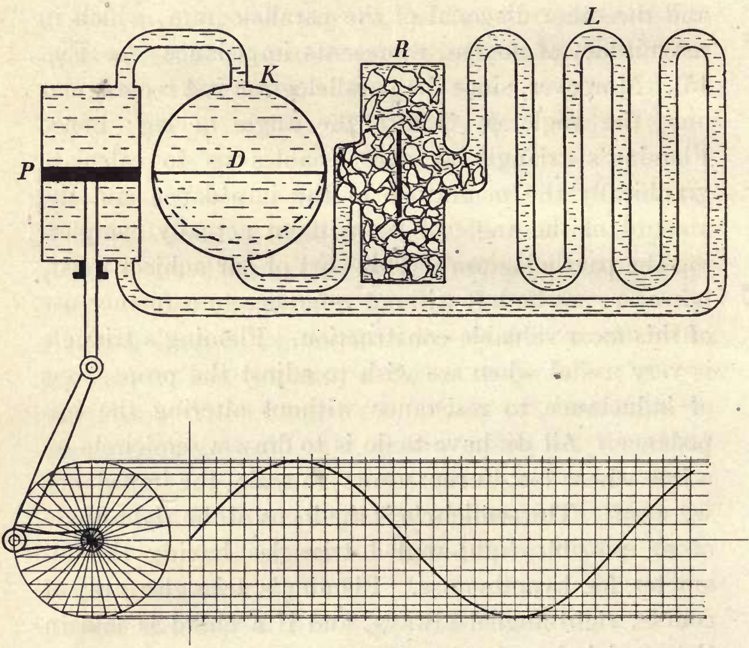

FIG. 16.

as the *capacity* of the circuit. Some reference was made to this property in Section 1* when dealing with the electrical circuit, and we now have to consider it rather more in detail, particularly as it affects an alternating current circuit. Precisely the same analogue as that adopted in Section 1* will suffice if, instead of the rotary pump there indicated, we substitute a reciprocating pump, as has been done in Fig. 16. The consideration of this figure shows two things, namely, that as the piston in the pump oscillates to and fro, the rush of water in and out of it at any particular instant will depend upon the *speed* of the piston and its speed again on its *position* at that instant, hence the rush of water into or out of the condenser will also be dependent upon the position of the piston. Now the connection between position, speed, and rate of change of speed is fortunately simple. Each is the rate of change of the last named, and if position follows a sine law, speed is a sine curve displaced backwards by a quarter period, and acceleration another sine curve again displaced backwards by 90 degrees. The rate of change of any sine curve can always be obtained by drawing another, lagging by 90 degrees.

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* "Outlines of Electrical Engineering."

To a deplorably large number of students there is little or, at most, a hazy difference between *quantity* and current; we must next, therefore, emphasise our definition of an electric current, or more properly speaking, our definition of what has to be considered the measure of magnitude of an electric current. What we measure when we measure a current is *a rate of flow of electricity*, and by carefully keeping this definition in view any difficulty in considering the effect of capacity in an alternating current circuit should be materially lessened. So far as the analogy goes, the water is *electricity*, its rate of flow is *current*, its volume is *quantity*, and its pressure is *potential*. The piston moves at its greatest speed just when it is in the middle of its stroke, and it will, therefore, be at this instant that the rate of flow into the condenser on the one side and out of it on the other will be greatest. In other words, we shall have at that instant the greatest current. At that instant also the diaphragm of the condenser will not be distended in any way. Expressing all these facts in electrical language, we can say that the current into a condenser supplied by an alternating E.M.F. is greatest at the commencement of the charge, and that at that same instant the back E.M.F. of the condenser is zero (since the diaphragm is unstretched). Since the diaphragm is unstretched no pressure is necessary on the part of the piston to drive the water in, if we neglect any friction in the pipes, but then, as the water enters and the diaphragm stretches, greater pressure is necessary on the part of the piston; also, as the piston advances towards the end of its stroke, producing distension of the diaphragm, its own speed and hence also the rush of water are diminishing, as we see from the curve. We may express these facts electrically by saying that when the current is greatest the E.M.F. driving it is nothing, and that as the back E.M.F. of the condenser rises so the E.M.F., to drive the current, rises, and at the same time the magnitude of the current falls off. Continuing our reasoning in this way, we see that

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* "Outlines of Electrical Engineering."

(corresponding to the end of the piston stroke) we reach a point where the two E.M.F.'s, namely, the forward and the back, are at their greatest, and the current is at its least, namely, nothing, and that matters then begin to reverse, the forward and the back E.M.F.'s diminishing and the current increasing, until at the instant when the condenser has just discharged, the current is at its greatest, and the two

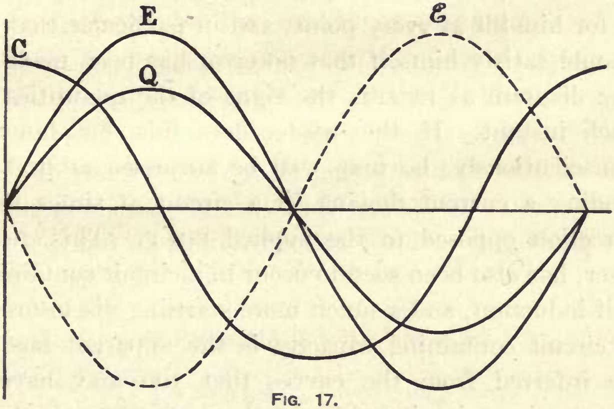

FIG. 17.

E.M.F's are both nothing, after which the whole cycle of operations reverses and then repeats. These facts have been represented by the curves in Fig. 17, and the curves will need no further remark to make them clear, beyond the statement that since the forward driving E.M.F. of the circuit and the back E.M.F. of the condenser are opposed to one another, it is, of course, necessary to represent the one as positive while the other is negative, and that when the current is flowing with the driving E.M.F. of the circuit it must be considered positive, and negative when against. The reader cannot be urged too strongly to reason out for himself very carefully the connection between the three quantities, forward E.M.F., back E.M.F., and current, at every stage of the operations described. It will be quite insufficient for a proper grasp of the matter for him merely to acquiesce in the diagram as shown. It is essential that he should carefully check it for himself at every point, and in particular that he should satisfy himself that no error has been made in the diagram as regards the signs of the quantities at each instant. If the reader does this for himself conscientiously, he may well be surprised at first at finding a current flowing in a circuit at times in a direction opposed to the applied E.M.F. This, however, has also been seen to occur in a circuit containing self-induction, and a much more startling disclosure in a circuit containing capacity is the apparent fact to be inferred from the curves that you may have a current in a circuit before you have an E.M.F. to drive it. This, indeed, needs explanation. The explanation is simple ; it is that the curves which we have been considering cannot be properly regarded as true of the circuit when the piston is just started, but only after it has been running for some little time. What really occurs at the very start is somewhat complicated, and the reader is referred to mathematical treatises for a consideration of this point. After a very few oscillations matters settle down to the relations indicated in our curves, and we do not need here to consider what occurs at the very start. The state of affairs indicated in our curves is generally summed up by saying that the current is in advance of the E.M.F. of the circuit, or in other words that it leads. In the case of the self-induction we have seen that the current was apparently behind the E.M.F., and in such cases it is said to lag.

MAGNITUDE OF CONDENSER CURRENT.

It is not difficult now to determine the magnitude of the condenser current. If we can settle the height of the crest of one of our sine curves it is all we need do. Obviously the easiest curve to deal with is the quantity curve. If K is the capacity in farads, then $Q = K\,V$, and therefore, as before, since $V = E_{max.} \sin. 2\,\pi\,f\,t$ or $E_{max.} \sin. p\,t$,

$$\frac{dQ}{dt} = p\,Q_{max.} \cos. p\,t = p\,K\,E_{max.} \cos. p\,t$$

$$\text{or,}\quad \left(\frac{dQ}{dt}\right)_{max.} = p\,K\,E_{max.}\ (\text{since } \cos._{max.} = 1)$$

$$\text{thus}\quad C_{max.} = p\,K\,E_{max.}$$

As to sign, we need only remember that Q is the quantity in the condenser, and, therefore, when Q is growing the current is *with* the applied E.M.F., and therefore positive.

As to the current here being "wattless," the remarks on pp. 56, etc., again apply. The E.M.F. of the circuit lags very, very nearly 90 degrees behind the current, but there must be a minute component in step with the current, overcoming the low resistance of the wires and condenser plates.

PERMITTANCE.

If we have in the circuit, instead of a coil of wire which possesses resistance and inductance, a condenser so constructed that the resistance of its plates is quite negligible, we have a circuit having a property not yet dealt with, namely, "permittance." We have seen above that the current flowing in and out of a condenser supplied with an alternating E.M.F. is given by the expression

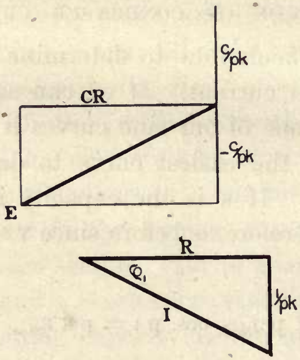

FIG. 18.

$$C_{max.} = p\,K\,E_{max.}$$

and hence we may say that the E.M.F. necessary to drive one ampère in and out of the condenser of capacity K with the frequency F is

$$\frac{1}{p\,K}, \text{ that is } \frac{1}{2\pi f\,K}$$

We can clearly treat this voltage as a property of the circuit in the same way that we did resistance and inductance, and since it is convenient to give it a name it has been given the name of "permittance." We also saw that the current in and out of the condenser was 90 degrees in advance of the E.M.F. required to drive it; that is to say, of the E.M.F. required to overcome the back E.M.F. of the condenser, and if we apply these facts to make a vector diagram, we shall get that shown in Fig. 18. Now it not infrequently occurs that we find a circuit possessed not only of resistance, and not only of inductance, but of resistance, inductance and permittance, and the question at once arises, how are we to calculate the current in a circuit possessed of these three properties when we are given the magnitude of the E.M.F. at work at its terminals? Fortunately the matter is very simple. Let us again consider what happens with a current of one ampère. If the current be one ampère we can draw at once three cranks, representing the resistance, inductance, and the permittance respectively, and since the inductance and permittance

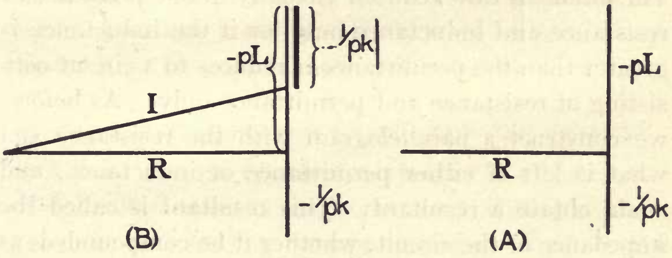

FIG. 19.

(Fig. 19A) are exactly opposite to one another, we can perform the operation of algebraic addition at once by graphical means. All we have to do is to shorten the greater of the two quantities by an amount equal to the lesser, as has been done in Fig. 19B. This is equivalent to saying that the permittance voltage precisely neutralises an equal amount of the inductance voltage, which may thus be regarded as being inoperative, leaving only the balance or remainder to be considered. The circuit is now reduced virtually to one possessed of resistance and inductance only, or if the inductance is greater than the permittance it reduces to a circuit consisting of resistance and permittance only. As before, we construct a parallelogram with the resistance and what is left of either permittance or inductance, and again obtain a resultant. This resultant is called the impedance of the circuit, whether it be compounded, as at first described, of resistance and inductance, or whether it be compounded of resistance and permittance, or whether, of course, it be compounded of all three as has just been shown. The case just described, in which the voltages required to overcome the three properties of the circuit have really been added vectorially, represents, of course, a circuit in which the three properties may be regarded as acting in series with one another, or we may analyse the circuit and separate its properties into three portions, and regard them as being connected in series with one another, as shown in the diagram. It is clear, of course, that when E.M.F.'s are in series they must be added, whereas if we have circuits in parallel with one another it is not the E.M.F.'s that have to be added to find the total, but the currents. If the circuits are in parallel there is only one E.M.F. operating on all at once, and when they are in series there is only one current flowing through them all at once. Vector diagrams, however, are at our disposal, and serve well for the elucidation of either case. All that is required is a little care in grasping the conditions of the circuit and precisely what we mean by our cranks. If the circuits are in series as above, the cranks represent E.M.F.'s. If they are in parallel our cranks must be used to represent currents.

CIRCUITS CONTAINING BOTH INDUCTION AND CAPACITY.

A very obvious enquiry in view of the fact that a self-induction causes a lagging current while a capacity gives rise to a leading current, arises as to whether it would not be possible to so arrange matters that the two effects neutralise one another. If we succeed in doing so it is clear that we shall have had to satisfy two conditions if one effect has to perfectly neutralise the other. We shall first have to see that the amount of lead is precisely equal to the amount of lag, and next we shall have to see that the magnitude of the leading current is equal to the magnitude of the lagging one. In point of fact it is quite easy, not only theoretically, but in practice, to arrange matters so that a self-induction shall be neutralised by a capacity. This is, however, only true in practice when the capacity demanded is not very great, large capacities being not only costly but difficult to construct. We have two distinct cases to consider—firstly, the case in which the self-induction and the capacity are in series with one another; and secondly, the case in which they are in parallel.

IN SERIES.

If we connect a non-inductive resistance, a pure self-induction and a pure capacity in series with one another, as in the diagram of Fig. 20,

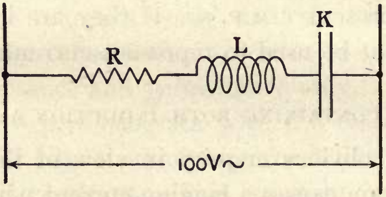

FIG. 20.

we have a case which is very simple to consider. Before, however, we proceed to its consideration, it may be as well to remark that it is, of course, immaterial in what order the three are connected, although it is impossible in practice to obtain a perfectly pure resistance and perfectly pure self-inductions and capacities. The occurrence of either or both of the other two properties in a portion of the circuit which we have supposed to contain only the third, is of no importance. For the sake of simplicity only have we shown three separate portions to the circuit. It is quite conceivable that an instrument should be constructed or a circuit made up so that the same wire or the same conductor introduces a resistance, self-induction and capacity in the path of the current. In such a case the accuracy of our result would be unimpaired, and indeed, as has been above suggested, every choking coil has some resistance, and practically all the so-called non-inductive resistances have some self-induction. In all common circuits the capacity present is entirely negligible. To return to our diagram: let us suppose a current of C ampères to be flowing

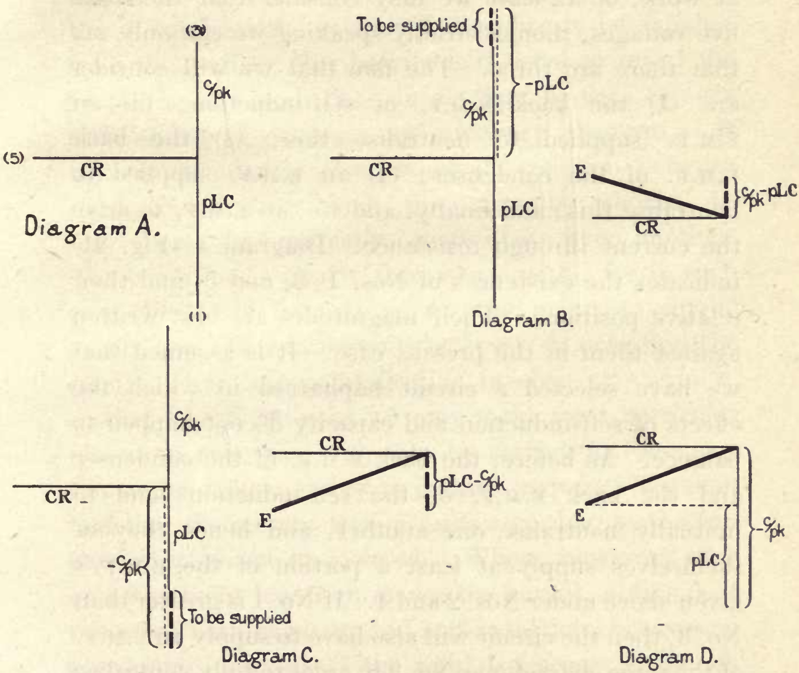

Diagram A.

Diagram B.

Diagram C.

Diagram D.

FIG. 21.

through this circuit. This we may represent by a crank of the right length, as is shown at C in Fig. 14. There will then be, if the current flows, five voltages at work, or at least we may consider that there are five voltages, though strictly speaking we can only say that there are three. The five that we will consider are (1) the back E.M.F. of self-induction; (2) an E.M.F. supplied to neutralise this; (3) the back E.M.F. of the condenser; (4) an E.M.F. supplied to neutralise this; and finally, and (5), an E.M.F. to drive the current through resistance. Diagram A (Fig. 21) indicates the existences of Nos. 1, 3, and 5, and their relative positions. Their magnitudes are also written against them in the present case. It is assumed that we have selected a circuit haphazard in which the effects of self-induction and capacity do not happen to balance. As before, the back E.M.F. of the condenser and the back E.M.F. of the self-induction tend to mutually neutralise one another, and hence they of themselves supply at least a portion of the E.M.F.'s given above under Nos. 2 and 4. If No. 1 is greater than No. 3, then the circuit will also have to supply an E.M.F. of the same character as No. 3 in order to fully neutralise No. 1; and on the other hand, if 3 is greater than 1, the circuit will have to assist 1 in order to perfectly neutralise 3. The amount of assistance which, so to speak, has to be given by the circuit either to 1 or 3, as the case may be, is readily determined graphically in the manner shown in diagrams B and C. Diagram B shows $p\,\mathrm{L}\,\mathrm{C}$ preponderating over the $\frac{\mathrm{C}}{p\,\mathrm{K}}$ crank, and the difference as shown had to be supplied from the mains, leaving the mains to supply altogether, when C R is included, the E.M.F. indicated at E. In diagram $C\ \frac{C}{pK}$ preponderated over $p\ L\ C$, with the result that a total E.M.F., as shown at E, has to be supplied from the mains. For brevity it is usual to insert the vectors as has been done in diagram D, thus at once arriving at E without an auxiliary diagram. It is now abundantly clear what the relation between the self-induction and the capacity should be for them to mutually neutralise one another. The condition is clearly that $p\ L\ C = \frac{C}{pK}$, for in such case there will be nothing for the mains to supply except C R. It is important, however, to note that the condition for this neutralisation involves the term p, so that any adjustment we may make for a particular circuit in this way will only be correct for one particular frequency, and at any other frequency we shall find that the current lags or leads according as the frequency is raised or lowered. When, however, such an adjustment has been made, the circuit, although it possesses both capacity and self-induction, behaves to alternating currents of the right frequency as though it had nothing but pure resistance. Unless, however, the co-efficient of self-induction is small it is an exceedingly difficult matter for all ordinary frequencies to provide a capacity sufficiently large to attain the desired effect. The amount of lag or lead is, of course, shown in every case by the position of the C R crank with reference to the E crank. The current must, as has already been pointed out, be in phase with the component of E.M.F. overcoming resistance.

IN PARALLEL.

When the capacity and self-induction were arranged in series with one another their function was to produce mutually neutralising E.M.F.'s. When, however, they are connected in parallel the E.M.F. at the terminals is at every instant determined by the E.M.F. of supply, and if the two are to neutralise one

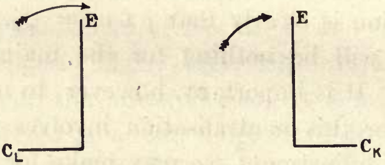

FIG. 22.

another it will be in respect of the current flowing; that is to say, they will, if properly proportioned, behave like an infinite resistance. This is to be understood from the following. Let E, in Fig. 22, represent the E.M.F. of supply, and the current that flows through a self-induction possessed of a negligible amount of resistance will be given by the crank C_L, whereas if the E.M.F. supplies a capacity only, the current will be indicated by a crank such as C_K. Now if the supply is made simultaneously between the same two points to both a self-induction and a capacity as in Diagram 23, we have cranks corresponding to C_L and C_K simultaneously present, and just as in the last case, where we had E.M.F.'s neutralising one another, so here we have currents neutralising one another. This will clearly be the case when $C_L = C_K$, and that is when

$$pL = \frac{1}{pK}$$

It is well to consider what the physical meaning of this is. The physical meaning is really very simple. It amounts to a circulation, or rather to an oscillation in the circuit formed by the self-induction and capacity, of a current which just keeps time with the pulsations of the E.M.F. in the supply mains. This current oscillating to and fro, as is indicated in the diagram, Fig. 23, gives rise at the terminals T_1, T_2, to an alternating E.M.F. which absolutely coincides in point of time with the E.M.F. of supply. Moreover, it coincides in magnitude so that no current at all can enter this self-contained circuit from the supply mains. It would appear then that the oscillation of this circuit may continue indefinitely, and this is the case provided always that our assumption that no resistance is pre-

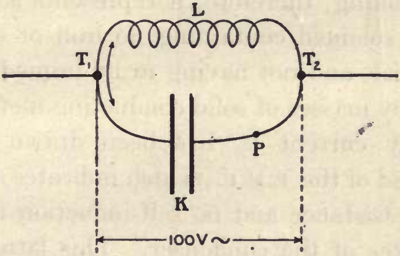

FIG. 23.

sent is true. This assumption has not been explicitly made above, but it has been implied, in that we drew the current C_L 90 degrees behind the E.M.F. Now if the current C_L is to lag by a full 90 degrees, it means two things: in the first place, that there is no resistance in the circuit, and in the second place that the magnetic field which gives rise to the self-induction is not carried by any material having hysteresis nor by any material in which any currents could be generated. Strictly speaking, therefore, it represents a resistanceless coil or solenoid containing no iron or other magnetic material, and not having in its immediate neighbourhood any masses of solid conducting metal. Again, the capacity current C_K has been drawn a full 90 degrees ahead of the E.M.F., which indicates an assumption of no resistance and no self-induction in the conducting plates of the condenser. This latter assumption is far more closely realised in practice than is the former. We further assume no dielectric hysteresis, and therefore no waste of power in the condenser dielectric. The failure of any of the assumptions made in regard to C_L or C_K would reduce the angle of lag to something less than 90 degrees, and in that case there would not be perect neutralisation of currents, but a small resultant would be present, as shown in Diagram 24. This small resultant is not 90 degrees out of phase with the E.M.F., and therefore represents some power supplied, and this power would, of course, go to make good the loss resulting from the heat generated in resistance, or in hysteresis, or in eddy currents, which, as we have just said, we cannot assume always to be entirely absent. Should they be entirely absent our statement first made above would be correct—namely, that the current will continue to oscillate in that self-contained circuit indefinitely. There being no heat produced, and no loss of energy, an analogy might be drawn between this and the

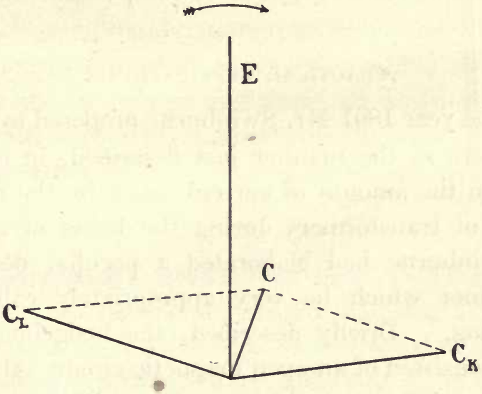

Fig. 24.

equally impossible case of a frictionless pendulum, making no friction with the air of course, and having no friction in the spring which carries it, oscillating continuously without diminution of amplitude. If we wish to introduce into this analogy the E.M.F. of supply, we can suppose underneath the pendulum a small rocking arm which, when the pendulum was stationary, served to start it swinging, but which rocks to and fro at precisely the same speed as that at which the pendulum oscillates; hence, although the top of the arm swings to and fro with the pendulum bob, it imparts no energy whatever to the pendulum bob, since the speeds of the two are identical at every instant, and no force can be exerted by the arm on the bob. The arm thus has no more work to do in oscillating to and fro than if the bob were not present, so also the E.M.F. of supply does no more work than if the self-induction and capacity were not there.

PRACTICAL APPLICATION.

In the year 1891 Mr. Swinburne proposed to employ condensers in the manner just described, in order to cut down the amount of current taken by the primary circuits of transformers during the hours of no-load. Mr. Swinburne had elaborated a peculiar design of transformer which he very appropriately called the "hedgehog." Briefly described, the hedgehog transformer consisted of an open magnetic circuit—that is, a circuit consisting only in part of iron, the rest of the magnetic path being in air. The iron part of the circuit was to all intents and purposes a bundle of iron wires round the middle of which coils were wound, the protruding ends of the iron wires being somewhat bent over. The object of this peculiar design was to reduce the weight of iron in the transformer, and so diminish the no-load losses which, as is shown elsewhere in this book,* occur almost exclusively in the iron. Such transformers demand a much higher exciting current, and although this current is, of course, a lagging one, the use of a number of such transformers on the mains would mean a considerable increase in the no-load current carried by them. The increase would be of the order of 50 per cent. over and above the currents demanded by similar transformers of the closed magnetic circuit type, and hence Mr. Swinburne suggested that condensers should be employed to reduce this. The dimensions of the condensers suitable for use in the manner mentioned to reduce the magnetising current supplied from the mains were so very large that the method found no commercial acceptance.

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* That is, the larger work, of which this is a portion published separately. See Preface.

RESONANCE.

The name "electric resonance" has been given to a form of the above phenomenon owing to its close analogy to resonance in acoustics. If we have the electric circuit consisting *entirely* of a self-induction and a capacity connected as in the diagram, Fig. 23, then that circuit possesses what is called a natural period of oscillation, just as a pendulum of given length or a string or organ pipe possess natural periods of oscillation. In the electrical case the natural period is given by the expression

$$\tau = 2\pi\sqrt{KL}$$

As described above, the natural period of the combination was just the same as the period of the applied E.M.F. If there were no resistance whatever—which we have assumed there is not—then if a surging could be started it would continue in the circuit with the frequency given by the periodic time. For example, if circuit were broken at the point P (Fig. 23), and the condenser charged from an external source in some way, and if then the ends of the broken circuit were again united with the turns of that coil, and when the magnetic field died down it would again charge the condenser, and this operation would repeat itself indefinitely; twenty times a second, if the periodic time of the circuit were $\frac{1}{10}$ of a second. If a further analogy is wanted to enable some conception of the parts played by the condenser and the self-induction respectively, one may regard the combination as being analogous to a flat steel spring, one end of which is clamped tightly in a vice, the other end being loaded with a weight. The stiffness of the spring will correspond to the capacity of the condenser, or rather, inversely to that capacity, so that a stiff spring will represent a condenser of small capacity, while a condenser of larger capacity will be represented by a weaker spring. The weight with which the free end of the spring is loaded corresponds to the inertia or self-induction of the circuit, the mass of the weight and the coefficient of self-induction being true analogues. Such an arrangement would have a natural period of oscillation, and if the free end of the spring be pulled to one side and then released, the weight would wag to and fro at a definite rate, the rate being obviously dependent upon both the stiffness of the spring and the mass of the weight. Now we may clearly treat this mechanical combination in two ways. We may on the one hand take hold of it firmly at the free end and pull it to and fro at any rate which pleases us, but not necessarily at a rate which pleases the combination, if we may use such an expression. There is only one rate which "pleases" the combination, that rate being, of course, its own natural rate, and if we take hold of the weight and pull it to and fro at any other rate of vibration, we impose upon the system what is called a forced rate of vibration. On the other hand, we may, if we are careful, supply impulses to the system which are carefully timed to agree with the rate of free vibration, so that whenever the weight is tending to move in one direction or the other, we are pushing or pulling it so as to help its natural tendency. The result of such treatment will be to increase the total amount of swing every time, and the swing will, therefore, increase more and more until the spring breaks. Resonance occurs in an electrical circuit then, if the natural period of oscillation of circuit, comprising a condenser and a self-induction, happens to agree precisely with the frequency of the impulses imparted to it by the alternating E.M.F. It is, of course, quite a remote contingency that such coincidence of frequency should occur very precisely, but it is not necessary for it to occur with great precision, since dangerous conditions arise if there be only approximate correspondence between the two frequencies, namely, the frequency of supply and the natural frequency. We might take as an example the case of a small high voltage alternate current generator designed for testing cables. The test is to take place on the dielectric, and the cable is for this purpose coiled in a tank of water, the two ends of the cable only standing out of the water. These two ends are carefully insulated, and one terminal of the alternator is connected to the copper, while the other terminal is connected to the water in the tank. Thus, when pressure is applied the whole of the dielectric is subjected to alternating stresses, and the arrangement simply becomes a condenser, of which the cable insulation is the dielectric, the copper of the cable is one coating, and the water in the tank the other. Now the armature of the alternator possesses self-induction in comparison with which resistance is negligible, and hence when the armature is connected to the cable the two have, as has been remarked above, a natural period of oscillation. It will be as well to give figures in this connection, and we will therefore suppose that the alternator armature has a co-efficient of self-induction of 0·6 henry, while the cable has a capacity of 2·4 microfarads. The natural period of this combination is 0·00754 of a second, and if, therefore, the alternator supplies the circuit with about this frequency, dangerous conditions will assuredly arise, and we shall have a breakdown. So far, however, nothing has been said as to how the condition may be dangerous, or why a breakdown results, except in the mechanical analogue, when it was stated that the swings would get greater and greater until the spring was broken. This assumed, in the mechanical case, that there was no friction of any kind; but, of course, although the friction may be small and of little importance for small swings, it becomes of considerable importance when large oscillations are taking place. The air friction, for example, at the centre of the swing, is a matter not to be neglected when the swings get very large; so, in the electrical case, the resistance of the various portions of the circuit may be small and unimportant for ordinary work, but when resonance occurs the resistance is the only thing which limits the oscillations, which apart from it would theoretically rise to an infinite magnitude. We saw, however, on page 74, that the effect of a self-induction and a capacity in series with one another was, if they were properly proportioned, to wipe one another out, and if an examination is made of the proportions which were then pointed out as being suitable, it will be seen that they are the very proportions which obtain when resonance is occurrent. In fact, we were in the former paragraph dealing with resonance in one of its forms.

Let us apply the results then obtained, namely, that the self-induction and capacity neutralise one another, to the present case. Taking the figures given above, and supposing that the armature has a negligible resistance and the cable a resistance of 2 ohms, we get the following results :—

$$p\,L \text{ must be equal to } {}^{1}/_{p\,K}$$

$$\therefore p^2 = \frac{1}{2{\cdot}4 \times {\cdot}6 \times 10^{-6}} = \frac{10^6}{1{\cdot}44.}$$

$$\therefore 2\pi f = \frac{10^3}{1{\cdot}2} \text{ and } f = 132{\cdot}5.$$

That is to say that if the alternator runs at a frequency of 132·5 the whole of its E.M.F. will be at liberty to cope with resistance only, and we have the result $C = E/R$. Now if $E = 5{,}000$ volts, as it well may for such tests, the current appears to be 2,500 ampères. We say "appears to be" advisedly, for the following reason. If the current did rise to 2,500 ampères the voltage at the terminals of the cable and alternator would be $p\,L\,C$ or $\frac{C}{p\,K}$, either expression giving the same result, of course, viz. 1,252,000 volts. This would never be reached, as the cable or alternator would simply break down first. If we enquire whence would come this huge force, the answer is that it comes from the synchronism of the impulses with the natural surgings, and corresponds to the force above described which breaks the spring in the vice. The effect of running at other frequencies may be calculated easily. If, for example, a frequency of 100 per second were supplied, $p\,L$ becomes 376·9, $1/p\,K$ is 664, impedance is 287, and the current about 17·4 ampères. The voltage at the generator terminals would be about 6,560 volts, and across the cable dielectric about 11,600.

VIRTUAL VALUES OF ALTERNATING QUANTITIES.

A decision of very great importance in connection with the alternating circuit has now to be considered. We have to decide precisely what is the value of a continually changing current when compared with a continuous current. If a man's income varied continually throughout the year, being nothing for the month of January, £10 for the month of February, £20 for March, and so on up to June, and then decreasing till it was nothing again for the month of December, it would be an extremely simple matter to determine what the annual income of that man was as compared with that of a man whose income was unvarying. If, on the other hand, two men whose incomes are to be compared are supposed to be engaged in an occupation such that the value of each £1 of income depended on the income itself, it would not be quite so simple. Suppose, for example, that an income of £10 in February was of less value to the man (say for speculative reasons) than an equal income in March, and that the money would be even more valuable in April, and so on, so that the actual value of a given sum varied throughout the year in exactly the same way as the man's income for the time being varied. In order to estimate properly the relative value of these two incomes we should have to go into a careful calculation. We have a very similar consideration to make in determining the relative values of an alternating and a continuous current. The value of a current to an engineer is, of course, the power that it represents, and it will be remembered that in the case of a continuous current the power of the current was proportional to the square of its value; in a word, that it was represented by the quantity $\text{C}^2\,\text{R}$. Now the only basis for the proper comparison of an alternating and continuous current is that of the power represented by each; hence what we have to compare is the total work which will be done by a continuous current of a given value in say, one second, with the total work that will be done by an alternating current of equal maximum value in the same time. The two cases are clearly shown in Fig. 25 for a current of one ampère. Most alternating currents oscillate from fifty to one hundred times a second, so that if we are to draw the

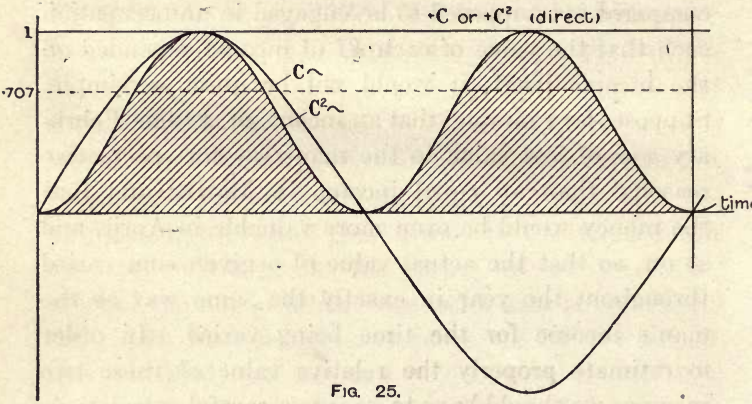

FIG. 25.

current for one second in our figure we shall have to show fifty to one hundred ripples. Clearly, then, it will be inconvenient to show the current for a whole second, and since all the ripples are exactly alike it will be far better to take the current for $\frac{1}{50}$ or $\frac{1}{100}$ of a second, so confining our attention to one complete ripple. Then, as all subsequent ripples are exactly like it, we have all the information we want. We have, however, to compare the work done in a given time, or the power, represented by these two currents, and this we have seen depends upon the squares of their values. We must, therefore, square these values at every instant of time, and if we do this it is clear that the straight line representing the continuous current will give us another straight line, for the square of its value gives us only one result. The full horizontal line, then, marked "$+ c$ or $+ c^2$ (direct)," will represent either a direct current of one ampère or that current squared. The squares of the alternating current values will depend at each instant on the corresponding values of the current, and since the square of a negative quantity gives us a positive quantity, a little reflection will show that the curve of squares of the alternating current is that shown by the wavy line marked $c^2_\sim$. Here, again, the reader cannot be too strongly urged to do this for himself. Let him construct a curve of sines as was advised on page 26, and let him make the length of his crank one inch, so that the height of the crest of his current ripple or wave will be one inch above the horizontal line: and finally let him evaluate c^2 at numerous points, and plot the curve of c^2. We now reason thus:—The work done in $\frac{1}{50}$ second by either current is measured by $c^2 \times t$; that is, by the area under the c^2 line. A glance at the diagram at once shows that the shaded area under the alternating c^2 is only half the total area under the direct c^2, for the unshaded wavy area is just the same as the shaded, and these two are both included under $+ c^2$. Hence the work done by a direct current of one ampère is double that done by a sinusoidal alternating current whose maximum is one ampère. Or we may say

$$\sim c^2\ t = \tfrac{1}{2}\ \text{direct}\ c^2 \times t,$$

$$\text{therefore} \sim c\ \text{corresponds to}\ \frac{1}{\sqrt{2c}}\ \text{(direct).}$$

It follows then that an alternating current with a maximum value of one ampère only does as much for us as a direct current of $\frac{1}{\sqrt{2}}$ or 0·707 ampère. The name given to this value of an alternating current is "virtual value," so that all virtual values (if sine curves) are 0·707 times the maxima. The names "effective" and "R.M.S." (meaning "root mean square") will also be met with.

The student may urge that the above reasoning only holds good for currents of one ampère, but he will see that this is not so, for it is after all only a question of scale. The same diagram would have done for a comparison and two currents (direct and maximum) of two ampères each, only we should have wanted double the scale for the currents, and four times the scale for their squares. If such scales were used there would be no difference between that diagram and the one we have given.

In our clock diagrams we have hitherto always made the lengths of our cranks represent maximum values. Here again it is merely a matter of changing the scale if we wish to make them represent virtual values. Since all measuring instruments are calibrated to read virtual values, and since we never need to deal with other than virtual values (except for analytical work, as when tracing out certain phenomena), we may at once make all our cranks stand for virtual values, and use the diagrams *as they are*, entirely unaltered. If a coil is traversed by a sinusoidal field, L is its co-efficient in *virtual volts*; and if E is virtual volts, then the current

$$C = \frac{E}{\sqrt{R^2 + p^2 L^2}} \textit{ virtual } \text{ampères.}$$

Also, if we have C virtual ampères at E virtual volts, the watts will be C × E × cos. ϕ.

The presence of the 2 in the denominator of the expression for watts at the foot of p. 50 is now explained. There we had $E_{max.}$ and $C_{max.}$, and each is to be divided by $\sqrt{2}$ to get virtual values; hence their product is divided by 2.

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